

Semi-simple group unification in the supersymmetric brane world

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Abstract

The conventional supersymmetric grand unified theories suffer from two serious problems, the large mass splitting between doublet and triplet Higgs multiplets, and the too long lifetime of the proton. A unification model based on a semi-simple group $SU(5)_{\text{GUT}} \times U(3)_{\text{H}}$ has been proposed to solve both of the problems simultaneously. Although the proposed model is perfectly consistent with observations, there are various mysteries. In this paper, we show that such mysterious features in the original model are naturally explained by embedding the model into the brane world in a higher dimensional space-time. In particular, the relatively small gauge coupling constant of the $SU(5)_{\text{GUT}}$ at the unification energy scale is a consequence of relatively large volume of extra dimensions. Here, we put the $SU(5)_{\text{GUT}}$ gauge multiplet in a 6-dimensional bulk and assume all fields in the $U(3)_{\text{H}}$ sector to reside on a 3-dimensional brane located in the bulk. On the other hand, all chiral multiplets of quarks, leptons and Higgs are assumed to reside on a 3-brane at a $\mathbf{T}^2/\mathbf{Z}_4$ orbifold fixed point. The quasi- $\mathcal{N} = 2$ supersymmetry in the hypercolor $U(3)_{\text{H}}$ sector is understood as a low-energy remnant of the $\mathcal{N} = 4$ supersymmetry in a 6-dimensional space-time. We further extend the 6-dimensional model to a 10-dimensional theory. Possible frameworks of string theories are also investigated to accommodate the present brane-world model. We find that the type IIB string theory with D3-D7 brane structure is an interesting candidate.

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I. INTRODUCTION

The supersymmetry (SUSY) is a very interesting symmetry which provides a natural explanation of the light Higgs scalar doublet in the standard model. That is, the SUSY-invariant mass term for the Higgs chiral multiplets can be suppressed by an appropriate chiral symmetry in the SUSY standard model, and if the breaking of the chiral symmetry is linked to the SUSY breaking, the masses of Higgs scalar doublets are naturally predicted at the SUSY-breaking scale (~ 1 TeV) [1]. However, we have to abandon this beautiful Giudice-Masiero mechanism [1] in SUSY grand unified theories (GUT's), since the Higgs doublets are necessarily accompanied by color-triplet Higgs multiplets whose masses should be at least of the order of unification scale $\sim 10^{16}$ GeV to account for the observed stability of proton. The chiral symmetry that forbids the SUSY-invariant mass term for the Higgs doublets also forbids the mass term for the Higgs color-triplets and their masses are also predicted at the SUSY-breaking scale inducing too rapid proton decay.

A SUSY unification model based on a semi-simple gauge group $SU(5)_{\text{GUT}} \times U(3)_H$ has been proposed to solve the above problem [2–4], in which the color-triplet Higgs multiplets acquire SUSY-invariant masses together with newly introduced colored chiral multiplets while the Higgs doublets remain massless. As a direct consequence of the Higgs structure this model solves also another problem in the SUSY standard GUT's; that is, the dangerous dimension-five operators [5] for the proton decay are suppressed by the symmetry that forbids the mass term for Higgs doublets. In this model the low-energy color group $SU(3)_C$ is a diagonal subgroup of $SU(3) \times SU(3)_H$ where the first $SU(3)$ is a subgroup of the $SU(5)_{\text{GUT}}$. The remarkable feature in this model is that the gauge coupling constants of the hypercolor group $U(3)_H$ should be very large, while the gauge coupling of the $SU(5)_{\text{GUT}}$ is relatively small at the unification scale M_{GUT} , so that we realize the approximate unification of three gauge coupling constants of the low-energy standard-model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

In this paper we embed this semi-simple unification model into the brane world in a higher dimensional space-time and show that the disparity of gauge coupling strengths is naturally understood in terms of relatively large volume of the extra dimensions. We put the $SU(5)_{\text{GUT}}$ gauge vector multiplet in a 6-dimensional bulk and assume that the extra 2-dimensional space is compactified on an orbifold $\mathbf{T}^2/\mathbf{Z}_4$. The standard quark, lepton and Higgs chiral multiplets are assumed to reside on a 3-dimensional brane (3-brane) at one

of the orbifold fixed points. We see that the $SU(5)_{\text{GUT}}$ gauge coupling receives a volume suppression due to the extra dimensions. On the other hand, we assume that the hypercolor $U(3)_H$ sector resides on a 3-brane located in the bulk (not at the fixed points). Thus, the gauge interactions of $U(3)_H$ are no longer suppressed. This configuration may also account for another mystery in the original $SU(5)_{\text{GUT}} \times U(3)_H$ model, namely the $\mathcal{N} = 2$ SUSY structure of the hypercolor $U(3)_H$ sector.

In our brane world the fundamental scale is determined as $M_* \simeq 10^{17}$ GeV for a successful phenomenology and the size of the compactified space is fixed as $\sim 10^{16}$ GeV so that the 4-dimensional Planck scale $M_{\text{Pl}} \simeq 2 \times 10^{18}$ GeV is obtained [6,7]. This lower-energy fundamental scale provides a solution to a potential problem in the original model that the gauge coupling constant of $U(1)_H$ is asymptotic non-free and it blows up around 6×10^{17} GeV below the Planck scale. However, this is not a problem in our brane world scenario, since the 4-dimensional Planck scale is merely an effective one and the present model is considered as only a low-energy description below the fundamental scale $M_* \simeq 10^{17}$ GeV of a more fundamental theory.

It is very attractive to consider that the SUSY is broken on a hidden 3-brane at another orbifold fixed point [8]. If it is the case, the present model is an extension of the gaugino-mediation model [9,10] of the SUSY breaking and provides a natural solution to the SUSY flavor problem. Here, the $SU(5)_{\text{GUT}}$ gaugino acquires a SUSY-breaking mass since they live in the 6-dimensional bulk and couples directly to the hidden-sector field. On the contrary, the $U(3)_H$ gauginos remain massless at the tree level, since they are localized on the 3-brane separated from the SUSY-breaking hidden brane. In this case we find an approximate GUT-unification of the masses of the SUSY standard-model gauginos, $m_{\tilde{G}_3} \simeq m_{\tilde{G}_2} \simeq m_{\tilde{G}_1}$, at the unification scale M_{GUT} . This result is very interesting, because the gaugino masses can be different from each others in the original semi-simple unification model [11].

In section II, we review briefly the SUSY $SU(5)_{\text{GUT}} \times U(3)_H$ unification model, and explain why the doublet-triplet mass splitting problem for Higgs multiplets is naturally solved. Here, we point out that there are various mysterious, but interesting features in the original model that are required for successful phenomenologies. In section III, we embed the $SU(5)_{\text{GUT}} \times U(3)_H$ in the brane world in a higher dimensional space-time. We explain, here, why we choose the dimension of extra space to be 2 and why the orbifold compactification is necessary. We consider the $\mathbf{T}^2/\mathbf{Z}_4$ orbifold as an example. We find that $\mathcal{N} = 4$ SUSY in the 6-dimensional bulk is crucial to have $\mathcal{N} = 2$ SUSY on the $U(3)_H$ 3-brane. We show

here that mysterious features in the original model are indeed naturally explained by the present embedding of the original model into the 6-dimensional space-time. In section IV, we discuss SUSY-breaking effects provided that the SUSY is broken on a hidden 3-brane at an orbifold fixed point. We discuss, in section V, a possible connection to string theories, since we consider that the string theories provide a natural framework of the brane world in a higher dimensional space-time. We find a preferable scheme may be provided by the type IIB string theory with D3-D7 branes. However, we also note that there are various unsolved problems in this string framework which may deserve further investigations. The last section is devoted to discussion and conclusions.

II. SUSY $SU(5)_{\text{GUT}} \times U(3)_{\text{H}}$ UNIFICATION MODEL

In this section, we discuss briefly a semi-simple unification model based on an $\mathcal{N} = 1$ SUSY $SU(5)_{\text{GUT}} \times U(3)_{\text{H}}$ gauge theory [2–4]. The $SU(5)_{\text{GUT}}$ is the usual GUT gauge group and its gauge coupling constant is in a perturbative regime, $\alpha_{\text{GUT}} \simeq 1/24$. The $U(3)_{\text{H}}$ is a hypercolor gauge group whose gauge interactions are sufficiently strong at the unification scale M_{GUT} as shown below. The usual quark and lepton chiral multiplets transform as $\mathbf{5}^*$ and $\mathbf{10}$ under the $SU(5)_{\text{GUT}}$ and they are all singlets of the hypercolor $U(3)_{\text{H}}$. A pair of Higgs multiplets H_k and \bar{H}^k ($k = 1, \dots, 5$) transforms as $\mathbf{5} + \mathbf{5}^*$ under the $SU(5)_{\text{GUT}}$ and as singlets under the $U(3)_{\text{H}}$.

In addition to the usual matter chiral multiplets we introduce six pairs of hyperquarks Q_α^ρ and \bar{Q}_ρ^α ($\alpha = 1, 2, 3; \rho = 1, \dots, 6$) which belong to $\mathbf{3} + \mathbf{3}^*$ of the hypercolor $SU(3)_{\text{H}}$ and have $U(1)_{\text{H}}$ charges 1 and -1 , respectively. Here, $U(3)_{\text{H}} \equiv SU(3)_{\text{H}} \times U(1)_{\text{H}}$. The first five pairs of Q_α^k and \bar{Q}_k^α ($\alpha = 1, 2, 3; k = 1, \dots, 5$) transform as $\mathbf{5}^* + \mathbf{5}$ under the $SU(5)_{\text{GUT}}$ and the last pair of Q_α^6 and \bar{Q}_6^α is a singlet of the $SU(5)_{\text{GUT}}$. To cause the desired breaking of the total gauge group $SU(5)_{\text{GUT}} \times U(3)_{\text{H}}$ down to the standard-model gauge group $SU(3)_{\text{C}} \times SU(2)_{\text{L}} \times U(1)_{\text{Y}}$, we furthermore introduce chiral multiplets X_β^α and X_0 which are an adjoint and a singlet representation of the $SU(3)_{\text{H}}$, respectively [4]. They would be regarded as $\mathcal{N} = 2$ SUSY partners of the vector multiplets of the hypercolor $U(3)_{\text{H}}$ as seen in the next section.

We now introduce a superpotential,

$$W = \lambda \bar{Q}_k^\beta X_\beta^\alpha Q_\alpha^k + \lambda' \bar{Q}_6^\beta X_\beta^\alpha Q_\alpha^6 + \kappa \bar{Q}_k^\alpha X_0 Q_\alpha^k + \kappa' \bar{Q}_6^\alpha X_0 Q_\alpha^6 - 3V^2 X_0. \quad (1)$$

We have the $\mathcal{N} = 2$ SUSY [3] in the hypercolor sector in the limit of $\lambda = \lambda' = \sqrt{2}g_{3\text{H}}$ and $\kappa = \kappa' = \sqrt{2}g_{1\text{H}}$, where $g_{3\text{H}}$ and $g_{1\text{H}}$ are gauge coupling constants of the $SU(3)_{\text{H}}$ and the

$U(1)_H$, respectively. We note here that the last term in eq.(1) corresponds to the Fayet-Iliopoulos (FI) F-term [12] and it is perfectly allowed by the $\mathcal{N} = 2$ SUSY. However, even if such an $\mathcal{N} = 2$ SUSY relation holds at the classical level, the $\mathcal{N} = 2$ SUSY in the hypercolor sector is explicitly broken by interactions with other sectors, since the SUSY of the total system is only $\mathcal{N} = 1$. For instance, quantum corrections from the $SU(5)_{\text{GUT}}$ gauge interactions change various coupling constants in the hypercolor $U(3)_H$ sector differently, loosing the $\mathcal{N} = 2$ relation among the Yukawa and gauge coupling constants. Thus, we do not impose such a restrictive $\mathcal{N} = 2$ SUSY condition in this paper. We see that the $\mathcal{N} = 2$ SUSY relation of coupling constants is not necessarily crucial for the following hypercolor dynamics.

As shown in ref. [4] we have a desired vacuum,

$$\langle Q_\alpha^r \rangle = \frac{V}{\sqrt{\kappa}} \delta_\alpha^r, \quad \langle \bar{Q}_r^\alpha \rangle = \frac{V}{\sqrt{\kappa}} \delta_r^\alpha, \quad \langle X_\beta^\alpha \rangle = \langle X_0 \rangle = \langle Q_\alpha^6 \rangle = \langle \bar{Q}_6^\alpha \rangle = 0. \quad (2)$$

Notice that this classical vacuum exists even at the quantum level [13,4]. It may be instructive to know that this vacuum is in a Higgs branch in an $\mathcal{N} = 2$ SUSY QCD [14] that may be stable against the present deviation from the $\mathcal{N} = 2$ to the $\mathcal{N} = 1$ SUSY. In this vacuum with $V \neq 0$ the total gauge group $SU(5)_{\text{GUT}} \times U(3)_H$ is broken down to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ and thus we take the vacuum-expectation value (vev) to be the unification scale $V \simeq M_{\text{GUT}} \simeq 10^{16}$ GeV. Here, the color $SU(3)_C$ is an unbroken diagonal subgroup of the $SU(3) \times SU(3)_H$ where the first $SU(3)$ is a subgroup of $SU(5)_{\text{GUT}}$, and the $U(1)_Y$ is a linear combination of a $U(1)$ subgroup of $SU(5)_{\text{GUT}}$ and the hypercolor $U(1)_H$. Thus, the gauge coupling constants α_3, α_2 and α_1 of the low-energy $SU(3)_C \times SU(2)_L \times U(1)_Y$ are given by

$$\alpha_3 \simeq \frac{\alpha_{\text{GUT}}}{1 + \alpha_{\text{GUT}}/\alpha_{3H}}, \quad \alpha_2 = \alpha_{\text{GUT}}, \quad \alpha_1 \simeq \frac{\alpha_{\text{GUT}}}{1 + \frac{1}{15}\alpha_{\text{GUT}}/\alpha_{1H}}, \quad (3)$$

where $\alpha_{3H} = g_{3H}^2/4\pi$ and $\alpha_{1H} = g_{1H}^2/4\pi$. We see that the GUT unification of the three gauge coupling constants α_3, α_2 and α_1 is approximately realized in a strong coupling region of the hypercolor gauge interactions, *i.e.* $\alpha_{3H}, \alpha_{1H} \gtrsim \mathcal{O}(1)$.

It is very important that massless fields of matter multiplets in the above vacuum eq.(2) are only a pair of hypercolor chiral multiplets, Q_α^6 and \bar{Q}_6^α , which now transforms as a color-triplet under the unbroken $SU(3)_C$. It is easy to give SUSY-invariant masses to these massless triplets by introducing the following superpotential,

$$W = h Q_\alpha^k \bar{Q}_6^\alpha H_k + \bar{h} Q_\alpha^6 \bar{Q}_k^\alpha \bar{H}^k. \quad (4)$$

The color-triplets Higgs H_a and $\bar{H}^a (a = 1, 2, 3)$ acquire masses of order of the unification scale $M_{\text{GUT}} \simeq V$ together with the sixth hyperquarks \bar{Q}_6^α and Q_α^6 in the vacuum eq.(2). On the other hand, the weak-doublet Higgs H_i and $\bar{H}^i (i = 4, 5)$ remain massless since there are no partners for them to form the SUSY-invariant masses. The Peccei-Quinn like chiral symmetry or the R-symmetry may forbid the tree-level masses for Higgs multiplets H_k and \bar{H}^k . The masslessness of the weak-doublets H_i and \bar{H}^i is guaranteed by such symmetries as long as they are unbroken. We find that only a discrete \mathbf{Z}_4 R-symmetry is a consistent symmetry to prevent the mass term for the Higgs multiplets.¹ We show the $\mathbf{Z}_{4\text{R}}$ charges for the chiral multiplets in Table I. It is now clear that the superpotentials eq.(1) and eq.(4) are consistent with this discrete $\mathbf{Z}_{4\text{R}}$ symmetry. (Notice that we have a continuous $\text{U}(1)_{\text{R}}$ in the limit of $h = 0$ [4,15].) We also easily find that the dimension-five operators for the proton decay [5] are forbidden by this $\mathbf{Z}_{4\text{R}}$ symmetry. Furthermore, the superpotential eq.(4) renders the vacuum eq.(2) to be a unique one in the theory and hence there are no massless moduli fields besides the SUSY standard-model particles [4].

We note that the above mechanism to give large masses to the color-triplet Higgs multiplets keeping massless Higgs doublets is very similar to the missing partner mechanism observed in the standard GUT [16]. However, there is a crucial difference. The masslessness of the Higgs doublets is guaranteed by the R-symmetry in the present model, while there is no consistent symmetry suppressing the SUSY-invariant Higgs mass in the missing partner model.

We comment on phenomenological problems in the original $\text{SU}(5)_{\text{GUT}} \times \text{U}(3)_{\text{H}}$ model. Since the Higgs multiplets giving masses for quarks and leptons are $\mathbf{5}$ and $\mathbf{5}^*$ of the $\text{SU}(5)_{\text{GUT}}$, we have the usual GUT relation,

$$m_b = m_\tau, \quad m_s = m_\mu, \quad m_d = m_e, \quad (5)$$

at the unification scale. This GUT relation is very successful for the third family, $m_b = m_\tau$, but it must be largely violated for the second and the first families. The possible lowest dimensional operators generating GUT-breaking effects in the quark- and lepton-mass matrices are

$$W = f_{ij} \mathbf{5}_i^* \mathbf{10}_j \bar{H} \frac{\langle Q \bar{Q} \rangle}{M_*^2}. \quad (6)$$

¹We show in section IV that the Higgs doublets acquire the SUSY-invariant mass of order of the gravitino mass after the $\mathbf{Z}_{4\text{R}}$ symmetry is broken down to the \mathbf{Z}_2 R-parity.

Here, M_* is the cut-off scale of the present theory. It is clear that if one takes $M_* \simeq M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV one obtains too small GUT-breaking effects as $|1 - m_s/m_\mu| \lesssim 10^{-2}$. This already implies that the cut-off scale should be much lower than the Planck scale M_{Pl} . In fact, to produce the observed mass spectrum for quarks and leptons we need [15],

$$\frac{\langle Q\bar{Q} \rangle}{M_*^2} \gtrsim \mathcal{O}(10^{-2}), \quad (7)$$

which leads to $M_* \lesssim 10^{17}$ GeV for $\langle Q\bar{Q} \rangle \simeq (10^{16} \text{ GeV})^2$.

As mentioned in the introduction, this lower-energy cut-off may solve another problem. That is, the GUT unification of the three gauge couplings, $\alpha_3 \simeq \alpha_2 \simeq \alpha_1$, to 5% accuracy requires sufficiently large $\text{U}(3)_{\text{H}}$ gauge coupling constants, with which the Landau pole of the hypercolor $\text{U}(1)_{\text{H}}$ gauge interactions appears at $\sim 6 \times 10^{17}$ GeV [17]. Thus, the presence of the cut-off M_* at $\sim 10^{17}$ GeV is obviously welcome to the present model.

We have shown, in this section, that somewhat mysterious, but interesting features are required in the original $\text{SU}(5)_{\text{GUT}} \times \text{U}(3)_{\text{H}}$ model for successful phenomenologies : the disparity of gauge coupling constants of $\text{SU}(5)_{\text{GUT}}$ and $\text{U}(3)_{\text{H}}$, the quasi- $\mathcal{N} = 2$ structure in the hypercolor $\text{U}(3)_{\text{H}}$ sector, and the relatively small cut-off scale M_* compared to the Planck scale M_{Pl} . We consider that such mysterious features are important indications of a more fundamental theory. In the next section we show that they are all naturally explained by embedding the original model into the brane world in a higher dimensional space-time.

III. EMBEDDING INTO THE BRANE WORLD

We embed the SUSY $\text{SU}(5)_{\text{GUT}} \times \text{U}(3)_{\text{H}}$ unification model discussed in the previous section into the brane world in a higher dimensional space-time. Before giving a detailed discussion we first determine the dimension n of the extra space.

Let us suppose that the fundamental theory is described in $3 + n$ dimensional space and a time. The Einstein-Hilbert action of gravity in $4 + n$ dimensional space-time is given by

$$\mathcal{S} = M_*^{2+n} \int \int \sqrt{-g^{(4+n)}} \mathcal{R} d^4x d^n y, \quad (8)$$

where M_* is the gravitational scale in the $4 + n$ dimensional space-time, and $g_{\mu\nu}^{(4+n)}$ and \mathcal{R} are the metric and the scalar curvature. We identify the gravitational scale in the $4 + n$ dimensional space-time, M_* , with the fundamental cut-off scale discussed in the previous section, and we take $M_* \lesssim 10^{17}$ GeV in the following discussion. The y denotes coordinates

of the extra space. The extra dimensions are assumed to be compactified with $\mathcal{V} = \tilde{L}^n$ the volume of the extra space. We assume the metric in the extra dimension to be orthogonal to those for our 4-dimensional space-time:

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + dy^2, \quad (9)$$

where $g(x)_{\mu\nu}$ is the metric in the 4-dimension. The integration over dy leads to the action in 4-dimension,

$$\mathcal{S}_4 = M_*^{2+n} \tilde{L}^n \int \sqrt{-g} \mathcal{R}_4 d^4x. \quad (10)$$

The coefficient in front of the integral must be the 4-dimensional Planck scale M_{Pl} so that

$$M_{\text{Pl}}^2 = M_*^2 (M_* \tilde{L})^n. \quad (11)$$

The Planck scale in the 4-dimensional space-time appears to be an effective scale, rather than a fundamental one [6,7].

From eq.(11) we obtain

$$10^{\frac{2}{n}} \lesssim \left(\frac{M_{\text{Pl}}}{M_*} \right)^{\frac{2}{n}} = (M_* \tilde{L}) = (M_{\text{Pl}} \tilde{L})^{\frac{2}{n+2}} \lesssim 10^{\frac{4}{n+2}}. \quad (12)$$

Here, the lower bound comes from the condition $M_* \lesssim 10^{17}$ GeV and the upper bound is obtained from the condition $\tilde{L}^{-1} \lesssim 1/M_{\text{GUT}}$.² The above equation (12) suggests $n \geq 2$. On the other hand, as the number of extra dimensions becomes larger, the effective compactification size \tilde{L} becomes smaller (see eq. (12)), which yields non-negligible contact interactions between sfermions on our brane and some SUSY-breaking field on a hidden brane causing too large flavour-changing neutral currents (FCNC's) [8] as shown in the next section. Thus, we are led to consider the safest case of $n = 2$, that is a 6-dimensional space-time. Here, the size of the extra dimensions is determined as $M_* \tilde{L} \simeq 10$ and thus $\tilde{L} \simeq 1/M_{\text{GUT}}$ and $M_* \simeq 10^{17}$ GeV. In the following discussion we concentrate ourselves on the 6-dimensional space-time.³ However, it may be straightforward to extend our analysis to different dimensional theories.

²If $\tilde{L} > 1/M_{\text{GUT}}$ we have Kaluza-Klein towers of adjoint matter multiplets below the unification scale, since we put later the $\text{SU}(5)_{\text{GUT}}$ vector multiplet in the bulk. They receive large GUT-breaking effects in their mass spectrum and hence they may affect significantly the gauge coupling unification.

³We will show, in section V, that our choice of the 6-dimensional space-time is naturally extended in a string theory.

We discuss first the group theoretical structure of 6-dimensional SUSY [18]. The 6-dimensional SUSY is specified by giving a pair of two integers $(\mathcal{N}_{4+}, \mathcal{N}_{4-})$. The integer $\mathcal{N}_{4+}(\mathcal{N}_{4-})$ represents the number of supercharges $\mathcal{Q}_{4+,A(4-,B)}^{(6)}$ that belong to $4_+(4_-)$ spinor representation of the $\text{SO}(5,1)$. The index $A(B)$ runs over $1, 2, \dots, 2\mathcal{N}_{4+}(\mathcal{N}_{4-})$. The pseudo-Majorana condition is imposed (see appendix A), and only half of them are independent:

$$\mathcal{Q}_{4+,A+\mathcal{N}_{4+}}^{(6)} = -\mathcal{Q}_{4+,A}^{(6)c} \quad (\text{for } A = 1, 2, \dots, \mathcal{N}_{4+}), \quad (13)$$

$$\mathcal{Q}_{4-,B+\mathcal{N}_{4-}}^{(6)} = -\mathcal{Q}_{4-,B}^{(6)c} \quad (\text{for } B = 1, 2, \dots, \mathcal{N}_{4-}). \quad (14)$$

Here, the $\mathcal{Q}^{(6)c}$ on the right-hand sides denotes charge conjugation of the $\mathcal{Q}^{(6)}$. For notations, see appendix A. The R-symmetry of $(\mathcal{N}_{4+}, \mathcal{N}_{4-})$ SUSY is $\text{Sp}(\mathcal{N}_{4+}) \times \text{Sp}(\mathcal{N}_{4-})$ and SUSY charges $\mathcal{Q}_{4+,A(4-,B)}^{(6)}$ belong to the fundamental representation of the $\text{Sp}(\mathcal{N}_{4+}(\mathcal{N}_{4-}))$.

By a dimensional reduction of the x_4 and x_5 directions, each supercharge is decomposed into two Weyl spinors in the following way:

$$\begin{aligned} \mathcal{Q}_{4-,B}^{(6)} &\rightarrow \begin{pmatrix} -\mathcal{Q}_\alpha^{(4)B+\mathcal{N}_{4-}} \\ -\bar{\mathcal{Q}}_B^{(4)\dot{\alpha}} \end{pmatrix}, \\ \mathcal{Q}_{4-,B+\mathcal{N}_{4-}}^{(6)} &\rightarrow \begin{pmatrix} \mathcal{Q}_\alpha^{(4)B} \\ -\bar{\mathcal{Q}}_{B+\mathcal{N}_{4-}}^{(4)\dot{\alpha}} \end{pmatrix}, \\ \mathcal{Q}_{4+,A}^{(6)} &\rightarrow \begin{pmatrix} \mathcal{Q}_\alpha^{(4)A+2\mathcal{N}_{4-}+\mathcal{N}_{4+}} \\ \bar{\mathcal{Q}}_{A+2\mathcal{N}_{4-}}^{(4)\dot{\alpha}} \end{pmatrix}, \\ \mathcal{Q}_{4+,A+\mathcal{N}_{4+}}^{(6)} &\rightarrow \begin{pmatrix} -\mathcal{Q}_\alpha^{(4)A+2\mathcal{N}_{4-}} \\ \bar{\mathcal{Q}}_{A+2\mathcal{N}_{4-}+\mathcal{N}_{4+}}^{(4)\dot{\alpha}} \end{pmatrix}, \end{aligned} \quad (15)$$

where $\mathcal{Q}_\alpha^{(4)a}(a = 1, \dots, 2\mathcal{N}_{4+} + 2\mathcal{N}_{4-})$ are 4-dimensional SUSY charges that belong to $(\mathbf{2}, \mathbf{1})$ spinor representation of the $\text{SO}(3,1)$. ($\bar{\mathcal{Q}}_a^{(4)\dot{\alpha}}(a = 1, \dots, 2\mathcal{N}_{4+} + 2\mathcal{N}_{4-})$ belong to $(\mathbf{1}, \mathbf{2})$ of the $\text{SO}(3,1)$.) Thus, the number of independent supercharges in the 4-dimensional space-time is $\mathcal{N} = 2\mathcal{N}_{4+} + 2\mathcal{N}_{4-}$. After the dimensional reduction, the $\text{SO}(2)_{45}$ rotation on the x_4 - x_5 plane, which was a subgroup of the $\text{SO}(5,1)$, now becomes an internal symmetry in the 4-dimensional theory. Since SUSY charges transform under the $\text{SO}(5,1)$ as spinor representations, they are charged under the $\text{SO}(2)_{45}$. Namely, $\mathcal{Q}_\alpha^{(4)a}(a = 1, \dots, 2\mathcal{N}_{4-})$ carry the $\text{SO}(2)_{45}$ charges -1 , while $\mathcal{Q}_\alpha^{(4)b+2\mathcal{N}_{4-}}(b = 1, \dots, 2\mathcal{N}_{4+})$ the charges $+1$.

We now consider the $\text{SU}(5)_{\text{GUT}}$ gauge vector multiplet that is supposed to live in the 6-dimensional bulk. We assume that the 6-dimensional space-time has an $\mathcal{N} = 4$ SUSY in the 4-dimensional sense before the compactification/orbifolding of the extra dimensional space.

We will see later that such a higher \mathcal{N} SUSY is necessary for having a quasi- $\mathcal{N} = 2$ structure in the hypercolor $U(3)_H$ sector. There are two choices, $(1, 1)$ and $(2, 0)$, for the SUSY in 6-dimension. If the extra 2-dimensional space is compactified, these two choices become the same $\mathcal{N} = 4$ SUSY. In the case of Abelian gauge theories, it is clear that the massless spectra after the compactification are identical to each other. Although we do not know the definition of the $(2, 0)$ theory for the non-Abelian case, it is expected that the $(2, 0)$ theory and the $(1, 1)$ theory are equivalent to each other via some duality similar to the T-duality in string theories [19], when the extra 2-dimensional space is compactified. Therefore, we take the $(1, 1)$ -SUSY theory in what follows. The R-symmetry of this 6-dimensional theory is $Sp(1)_{4+} \times Sp(1)_{4-} \simeq SU(2)_{4+} \times SU(2)_{4-} \simeq SO(4)_R$.

The vector multiplet is a unique supermultiplet of the $(1, 1)$ SUSY theory besides the gravity multiplet. It consists of a 6-dimensional vector field A_μ ($\mu = 0, \dots, 5$), two pseudo-Majorana-Weyl spinors with opposite chirality $\Lambda_{4+,B}$ and $\Lambda_{4-,A}$ ($A, B = 1, 2$)⁴ and two complex scalar fields σ' and σ'' . All are certainly adjoint representations of the $SU(5)_{GUT}$. The vector field A_μ is an invariant under the R-symmetry $SU(2)_{4+} \times SU(2)_{4-}$. The spinor $\Lambda_{4-,A}$ transforms as a doublet of the $SU(2)_{4+}$ and the other spinor $\Lambda_{4+,B}$ as a doublet of the $SU(2)_{4-}$. Four real components of scalar fields belong to $(\mathbf{2}, \mathbf{2})$ representation of the R-symmetry, which transforms as

$$\begin{pmatrix} \sigma' & \sigma'' \\ \sigma''^* & -\sigma'^* \end{pmatrix}_{BA} \rightarrow SU(2)_{4-,B}{}^{B'} \begin{pmatrix} \sigma' & \sigma'' \\ \sigma''^* & -\sigma'^* \end{pmatrix}_{B'A'} SU(2)_{4+,A}{}^{A'}. \quad (16)$$

In other words, they form a vector of the $SO(4)_R \simeq SU(2)_{4+} \times SU(2)_{4-}$.

If the extra 2-dimensional space is compactified on a torus \mathbf{T}^2 , we have an $\mathcal{N} = 4$ SUSY theory in the 4-dimensional space-time; the flat metric of torus preserves all the SUSY. This 4-dimensional theory contains an $\mathcal{N} = 4$ gauge vector multiplet consisting of a 4-dimensional vector field A_i ($i = 0, \dots, 3$), four fermion partners $\chi_{\alpha,a}$ ($a = 1, \dots, 4$) and three complex scalar fields $\sigma, \sigma', \sigma''$. The four Weyl fermions $\chi_{\alpha,a}$ are obtained from the 6-dimensional fermion fields $\Lambda_{4+(\mathbf{4}_-)}$ by a decomposition similar to eq.(15) as

$$\Lambda_{4+,1} \rightarrow \begin{pmatrix} -i\chi_{\alpha,1} & \\ & -i\bar{\chi}^{\dot{\alpha},2} \end{pmatrix} \quad \Lambda_{4-,1} \rightarrow \begin{pmatrix} & -i\chi_{\alpha,3} \\ -i\bar{\chi}^{\dot{\alpha},4} & \end{pmatrix}, \quad (17)$$

and the complex σ field is constructed by the original 6-dimensional vector field as

⁴ This means that the Weyl spinors, $\Lambda_{4+,B}$ and $\Lambda_{4-,A}$ ($A, B = 1, 2$), satisfy similar relations to eq.(13) and eq.(14), respectively.

$$\sigma(x, y) \equiv \frac{1}{\sqrt{2}}(A_4(x, y) + iA_5(x, y)). \quad (18)$$

In terms of $\mathcal{N} = 1$ SUSY multiplets, the $\mathcal{N} = 4$ vector multiplet consists of one vector multiplet $\mathcal{W}_\alpha = (i\chi_1, F_{ij})$ and three chiral multiplets $\Sigma = (i\sigma, \chi_2)$, $\Sigma' = (i\sigma', \chi_3)$ and $\Sigma'' = (i\sigma'', \chi_4)$. Thus, we have three adjoint chiral multiplets. If they exist as massless particles, they change the renormalization-group-equation (RGE) flow of the standard-model gauge coupling constants, leading to a blow up of gauge couplings well below the unification scale M_{GUT} . This too many adjoint multiplets is a direct consequence of the $\mathcal{N} = 4$ SUSY in 4-dimension, and therefore, we do not want the $\mathcal{N} = 4$ SUSY to be preserved below the compactification scale \tilde{L}^{-1} (\sim the unification scale M_{GUT}).

To obtain the $\mathcal{N} = 1$ SUSY theory we have to do some orbifolding [20] by using a discrete subgroup of the symmetry $\text{SO}(2)_{45} \times \text{SU}(2)_{4+} \times \text{SU}(2)_{4-}$.⁵ Without specifying a concrete form of the orbifold group, we can show that only the $\mathcal{N} = 1$ vector multiplet \mathcal{W}_α remains massless after the orbifolding as follows. SUSY charges and/or bulk fields that are invariants of the orbifold-group action survive the orbifolding. That is, SUSY charges that are charged under the action disappear at low energies and bulk particles charged under the action do not have massless modes in their Kaluza-Klein spectra, as we see later. Suppose that we have an orbifolding after which we have only one SUSY charge left unbroken among four. Then, only one fermion is left massless after the orbifolding, because the fermion fields $\bar{\chi}^a$ transform under the the orbifolding symmetry in the identical way to the supercharges \mathcal{Q}^a . This massless fermion must belong to the vector multiplet $\mathcal{W}_\alpha = (i\chi_1, F_{ij})$ because the vector field A_i is invariant under the symmetry. On the other hand, all scalar fields σ , σ' and σ'' are projected out along with the three fermion partners of the unbroken $\mathcal{N} = 1$ SUSY. Therefore, if we keep the $\mathcal{N} = 1$ SUSY after the orbifolding, then only the vector multiplet \mathcal{W}_α remains massless and we do not have to worry about the emergence of unwanted adjoint chiral multiplets.

Let us see how the above procedure works, taking an explicit example of the orbifolding. The orbifolding is, in terms of the 6-dimensional theory, given by gauging a symmetry which is a combination of space (x_4 - x_5) rotation and reflection (*i.e.* $\text{O}(2)_{45}$) and internal symmetry transformations. After the compactification (dimensional reduction), however, the space

⁵This orbifolding is also necessary to obtain chiral representations $\mathbf{5}^* + \mathbf{10}$ in SUSY standard-model sector.

symmetry of the compactified 2-dimensional space is treated as an internal symmetry, or more specifically, as an $O(2)_{45}$ R-symmetry. Thus, the orbifold group is considered as a subgroup of purely internal symmetries. We will seek for a suitable orbifold group using the 4-dimensional terms, because of the convenience.

We now search for a candidate for the orbifold group which is a subgroup of the symmetry group (space-rotation symmetry $SO(2)_{45}$) \times (6-dimensional R-symmetry $SU(2)_{4+} \times SU(2)_{4-}$). This group is embedded into the maximally possible R-symmetry $SU(4)_R$ of the 4-dimensional $\mathcal{N} = 4$ SUSY. In the following analysis, we consider the whole $SU(4)_R$ group.⁶ Notice that as long as we take the orbifold group within the maximal torus of the $SU(4)_R$, which is the case for the orbifolding we adopt in this paper, the orbifold group is always contained in the original $SO(2)_{45} \times SU(2)_{4+} \times SU(2)_{4-}$, since the maximal torus of the $SU(4)_R$ is the same as that of the original group.⁷ The SUSY charges $Q^{(4)a}$ transform as $\mathbf{4}$ under the $SU(4)_R$, the fermions χ_a as $\mathbf{4}^*$ and the scalars

$$\sigma_{ab} \equiv \begin{pmatrix} 0 & \sigma & \sigma' & \sigma'' \\ -\sigma & 0 & \sigma''^* & -\sigma'^* \\ -\sigma' & -\sigma''^* & 0 & \sigma^* \\ -\sigma'' & \sigma'^* & -\sigma^* & 0 \end{pmatrix} \quad (19)$$

as the 2nd rank anti-symmetric tensor of $\mathbf{4}^*$ (*i.e.* $\wedge^2 \mathbf{4}^*$). Here, the $SO(2)_{45} \times SU(2)_{4+} \times SU(2)_{4-}$ are embedded in the $SU(4)_R$ as

$$\begin{pmatrix} e^{-i\frac{\varphi}{2}} & & & \\ & e^{-i\frac{\varphi}{2}} & & \\ & & e^{i\frac{\varphi}{2}} & \\ & & & e^{i\frac{\varphi}{2}} \end{pmatrix}, \begin{pmatrix} SU(2)_{4-} & \\ & SU(2)_{4+} \end{pmatrix} \subset (SU(4)_R), \quad (20)$$

where $0 \leq \varphi < 4\pi$.

Let us consider an $SU(3)_R$ subgroup of the $SU(4)_R$,

$$\begin{pmatrix} 1 & \\ & SU(3)_R \end{pmatrix} \subset (SU(4)_R). \quad (21)$$

⁶ There are two reasons why we proceed our discussion in terms of $SU(4)_R$. (1) The space symmetry $SO(2)_{45}$ can be treated just as an internal R-symmetry. (2) Relations between the 6-dimensional R-symmetry $SU(2)_{4+} \times SU(2)_{4-}$ and the R-symmetry $U(1)_R \times SU(2)_R \times SU(2)_F$ in the hypercolor $U(3)_H$ sector, which appears later, can be explicitly described in terms of $SU(4)_R$. Discussion in section V gives a geometrical explanation of the relations among various R-symmetries.

⁷ In fact, it is not always necessary that the orbifold group be contained in the maximal torus of the $SO(2)_{45} \times SU(2)_{4+} \times SU(2)_{4-}$.

If the orbifold group is contained in this $SU(3)_R$ subgroup, then the SUSY charge $\mathcal{Q}^{(4)1}$ is invariant under the orbifold group, and hence the $\mathcal{N} = 1$ SUSY survives the orbifolding. We take a \mathbf{Z}_4 subgroup of the $SU(3)_R$ as the orbifold group⁸ whose generator is

$$\begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & -1 \end{pmatrix} \in (SU(4)_R). \quad (22)$$

This \mathbf{Z}_4 subgroup is in the maximal torus of the $SU(3)_R$, and hence it is contained in the $SO(2)_{45} \times SU(2)_{4+} \times SU(2)_{4-}$. This generator is decomposed into a product of $SO(2)_{45}$ and $SU(2)_{4+} \times SU(2)_{4-}$ elements as:

$$\begin{pmatrix} 1 & & & \\ & i & & \\ & & i & \\ & & & -1 \end{pmatrix} = \begin{pmatrix} e^{i\frac{\pi}{4}} & & & \\ & e^{i\frac{\pi}{4}} & & \\ & & e^{-i\frac{\pi}{4}} & \\ & & & e^{-i\frac{\pi}{4}} \end{pmatrix} \begin{pmatrix} e^{-i\frac{\pi}{4}} & & & \\ & e^{i\frac{\pi}{4}} & & \\ & & e^{i\frac{3\pi}{4}} & \\ & & & e^{-i\frac{3\pi}{4}} \end{pmatrix}. \quad (23)$$

Therefore, the above orbifolding is equivalent to the identification of extra-dimensional space \mathbf{T}^2 under the $-\pi/2$ rotation⁹ in the x_4 - x_5 plane accompanied by a suitable twist under the internal $SU(2)_{4+} \times SU(2)_{4-}$ transformation. The $SU(3)_R$ action on the fermions and the scalars are given by

$$\begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & \\ & SU(3)_R^* \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{pmatrix}, \quad (24)$$

$$\begin{pmatrix} \sigma \\ \sigma' \\ \sigma'' \\ \sigma^* \\ \sigma'^* \\ \sigma''^* \end{pmatrix} \rightarrow \begin{pmatrix} SU(3)_R^* & \\ & SU(3)_R \end{pmatrix} \begin{pmatrix} \sigma \\ \sigma' \\ \sigma'' \\ \sigma^* \\ \sigma'^* \\ \sigma''^* \end{pmatrix}. \quad (25)$$

Now, it is clear that we can eliminate the unwanted σ and χ bulk fields from massless spectrum by the \mathbf{Z}_4 orbifolding of the torus \mathbf{T}^2 . This is because wave functions of the \mathbf{Z}_4

⁸This \mathbf{Z}_4 group is taken just as an example. There are many other examples of orbifold group that satisfy phenomenological requirements.

⁹ Note that the $\sigma \propto (A_4 + iA_5)$ transforms as $\sigma \rightarrow e^{i\varphi}\sigma$ under the $SO(2)_{45}$ when the $SO(2)_{45}$ element is represented as $\text{diag}(e^{-i\varphi/2}, e^{-i\varphi/2}, e^{i\varphi/2}, e^{i\varphi/2})$ on the $\mathbf{4}$ representation of the $SU(4)_R$.

charged particles in the bulk must vanish at the orbifold fixed points: Zero-modes in their Kaluza-Klein towers are eliminated and their lowest-energy states have Kaluza-Klein masses of order the $\tilde{L}^{-1} \simeq M_{\text{GUT}}$ disappearing from the low-energy spectrum. Therefore, we have an $\mathcal{N} = 1$ SUSY theory in the bulk with only an $\text{SU}(5)_{\text{GUT}}$ gauge vector multiplet below the Kaluza-Klein mass scale $\tilde{L}^{-1} \simeq M_{\text{GUT}}$.

We adopt, in most of this paper, the above \mathbf{Z}_4 orbifolding. This means that the torus \mathbf{T}^2 in the x_4 - x_5 space directions must have the \mathbf{Z}_4 rotational symmetry. Therefore, the \mathbf{T}^2 torus is the quotient of \mathbf{R}^2 by the square lattice, as shown in Fig.1. The $\mathbf{T}^2/\mathbf{Z}_4$ orbifold has two distinct \mathbf{Z}_4 fixed points F_v and F_h .¹⁰ We put a 3-brane at one of the fixed points F_v and assume that the chiral multiplets of quarks, leptons and a pair of Higgs H_k and \bar{H}^k ($k = 1, \dots, 5$) all reside on this 3-brane. The low-energy theory on the 3-brane is described by the 4-dimensional $\mathcal{N} = 1$ SUSY theory, where the SUSY is generated by one of the four SUSY charges, namely the $\mathcal{Q}^{(4)1}$, since the other three SUSY charges are not invariant under the \mathbf{Z}_4 orbifold group transformation.

We now show that the $\text{SU}(5)_{\text{GUT}}$ gauge coupling g_{GUT} in the 4-dimensional space-time receives a volume suppression. The gauge coupling in the fundamental theory is defined as

$$\mathcal{S} = M_*^2 \int d^4x \int_{\mathbf{T}^2/\mathbf{Z}_4} d^2y \int d^2\theta \sqrt{-g^{(6)}} \frac{1}{g_0^2} \mathcal{W}_\alpha \mathcal{W}^\alpha, \quad (26)$$

where \mathcal{W}_α and g_0 are the field strength of the vector multiplet and the dimensionless gauge coupling of the fundamental theory, respectively. The integration of d^2y yields a 4-dimensional action as

$$\mathcal{S}_4 = \frac{1}{4} (M_* L)^2 \int d^4x \int d^2\theta \sqrt{-g} \frac{1}{g_0^2} \mathcal{W}_\alpha \mathcal{W}^\alpha, \quad (27)$$

where L is the lattice spacing and $1/4$ the volume of $\mathbf{T}^2/\mathbf{Z}_4$ measured in the unit of L^2 (*i.e.* $\mathcal{V} = \tilde{L}^2 = L^2/4$). Thus, the 4-dimensional gauge coupling defined as $\alpha_{\text{GUT}} \equiv 4\alpha_0 (M_* L)^{-2}$ has a suppression factor $4/(M_* L)^2 \simeq 10^{-2}$. The experimental value $\alpha_{\text{GUT}} \simeq 1/24$ gives the gauge coupling $\alpha_0 \equiv g_0^2/4\pi \simeq 4$. This suggests that the fundamental theory is strong entirely, but as shown in [10] it may be still weak enough to perform a perturbative analysis. Above the compactification scale (*i.e.* the unification scale M_{GUT}), Kaluza-Klein towers including excitations of the adjoint chiral multiplets Σ , Σ' and Σ'' contribute in general

¹⁰To be precise, $\mathbf{T}^2/\mathbf{Z}_4$ orbifold has another singularity, which is not fixed under the \mathbf{Z}_4 but fixed under the $\mathbf{Z}_2 \subset \mathbf{Z}_4$.

to RGE's for the 4-dimensional gauge coupling $\alpha_{\text{GUT}}(\mu)$, where μ is the renormalization energy scale [21]. However, the Kaluza-Klein towers form $\mathcal{N} = 4$ SUSY multiplets and their contributions to the RGE's are canceled out completely and vanish at all orders of the perturbation theory. Thus, we have the usual RGE's in the 4-dimensional space-time and the coupling runs logarithmically. The difference between $\alpha_{\text{GUT}}(\mu = M_*)$ and $\alpha_{\text{GUT}}(\mu = M_{\text{GUT}})$ is quite small, and α_0 at the fundamental scale $M_* \simeq 10^{17}$ GeV is about the same as that at the unification scale $\mu = M_{\text{GUT}}$.

Now, we discuss SUSY in the hypercolor $U(3)_{\text{H}}$ sector which is supposed to reside on a 3-brane in the 6-dimensional bulk (see Fig.1). We have stated previously that we need an $\mathcal{N} = 4$ SUSY in the bulk theory (after a reduction to the 4 dimensions) for having naturally the quasi- $\mathcal{N} = 2$ SUSY structure in the hypercolor sector. Indeed, the $U(3)_{\text{H}}$ 3-brane has tension, and because of this energy density, not all of (at most half of) the SUSY charges in the bulk theory is left unbroken on the $U(3)_{\text{H}}$ -sector fields (see appendix B for details). Therefore, $(0, 1)$ SUSY in the bulk, which would become the 4-dimensional $\mathcal{N} = 2$ SUSY after the dimensional reduction, is not enough for our purpose. This is the reason why we need one more SUSY in the original 6-dimensional bulk theory. We assume half of the $\mathcal{N} = 4$ (*i.e.* $\mathcal{N} = 2$) SUSY to be realized on the hypercolor 3-brane, taking the $(1, 1)$ SUSY in the 6-dimensional bulk. We show in section V that this is indeed the case in string theories.

Although the orbifolding reduces the $\mathcal{N} = 4$ SUSY in the bulk theory to the $\mathcal{N} = 1$ SUSY in the effective 4 dimensional theory, the SUSY realized on the 3-brane alone is not affected by the orbifolding. This is because the $U(3)_{\text{H}}$ -sector fields are confined on the 3-brane and do not see the global structure of the x_4 - x_5 space but see only the local neighbourhood around the 3-brane. The discrete action of the orbifold group relates the 3-brane to its mirror images. We can choose wave functions on those mirror-image 3-branes so that the local structures around those 3-branes are all equivalent to each other under the orbifold-group action. Thus, we see that there is no effect of the orbifolding as long as we are concerned only in the physics on the 3-brane (*i.e.* physics of the $U(3)_{\text{H}}$ vector multiplets and $Q^i, \bar{Q}_i, Q^6, \bar{Q}_6$ chiral multiplets).

The breaking of P_4 and P_5 gives rise to Nambu-Goldstone (NG) bosons, which correspond to fluctuation modes of the $U(3)_{\text{H}}$ 3-brane. They are always massless if the parallel transport symmetry is broken only by the presence of the 3-brane. However, the symmetry is already broken by the orbifolding of the torus. Therefore, we can expect that the NG bosons get masses and the position of the brane is stabilized by some yet unknown dynamics (see also

the discussion in section V). We assume throughout this paper that it is indeed the case.¹¹

The field theory on the $U(3)_H$ 3-brane alone is, therefore, described by an $\mathcal{N} = 2$ SUSY theory in the 4-dimensional space-time. Here, we have $\mathcal{N} = 2$ vector multiplets of the $U(3)_H$ gauge theory and six hypermultiplets $(Q_\alpha^\rho, \bar{Q}_\rho^\alpha)$ ($\alpha = 1, 2, 3; \rho = 1, \dots, 6$). The $\mathcal{N} = 2$ vector multiplets are decomposed into an $\mathcal{N} = 1$ vector multiplet $\mathcal{W}_\alpha^{SU(3)_H}$ of the $SU(3)_H$ and a vector multiplet $\mathcal{W}_\alpha^{U(1)_H}$ of the $U(1)_H$, and $\mathcal{N} = 1$ chiral multiplets X_β^α and X_0 . We have a superpotential eq.(1) in the previous section with the $\mathcal{N} = 2$ SUSY coupling relation:

$$\lambda = \lambda' = \sqrt{2}g_{3H}, \quad \kappa = \kappa' = \sqrt{2}g_{1H}. \quad (28)$$

The $\mathcal{N} = 2$ SUSY on this brane is, however, broken by gauge interactions of the bulk $SU(5)_{GUT}$ since the bulk SUSY is only $\mathcal{N} = 1$. Thus, the above Yukawa coupling constants, λ, λ' and κ, κ' , receive renormalization effects differently from the $SU(5)_{GUT}$ gauge-multiplet loops. And the $\mathcal{N} = 2$ SUSY relation eq.(28) survives no longer at low energies. However, it is extremely interesting that the presence of the $U(3)_H$ adjoint chiral multiplets X_β^α and X_0 is an automatic consequence of the theory. Recall that we have introduced them, by hand, to cause the desired symmetry breaking in the original model.

We turn to discuss R-symmetry on the hypercolor 3-brane alone. The possible maximum R-symmetry of the 4-dimensional $\mathcal{N} = 2$ SUSY is $U(1)_R \times SU(2)_R \times SU(2)_F$, which is a subgroup of the $SU(4)_R$ as

$$\begin{pmatrix} e^{i\varphi/2} & & & \\ & e^{-i\varphi/2} & & \\ & & e^{-i\varphi/2} & \\ & & & e^{i\varphi/2} \end{pmatrix}, \begin{pmatrix} * & & * \\ & SU(2)_F & \\ * & & * \end{pmatrix} \subset (SU(4)_R), \quad (29)$$

where $0 \leq \varphi < 4\pi$, and the $*$'s in the four corners of the 4×4 matrix represent the $SU(2)_R$. As shown in the appendix B, one SUSY charge from $\mathcal{Q}^{(4)1}$ and $\mathcal{Q}^{(4)2}$ and one from $\mathcal{Q}^{(4)3}$ and $\mathcal{Q}^{(4)4}$ form the $\mathcal{N} = 2$ SUSY, and in the above choice of the embedding of the $U(1)_R \times SU(2)_R \times SU(2)_F$ into the $SU(4)_R$, they are $\mathcal{Q}^{(4)1}$ and $\mathcal{Q}^{(4)4}$. Notice that the $SU(2)_F$ is not R-symmetry acting on the relevant $\mathcal{N} = 2$ SUSY charges $\mathcal{Q}^{(4)1}$ and $\mathcal{Q}^{(4)4}$, but a flavour symmetry that acts only on the $SU(5)_{GUT}$ adjoint chiral multiplets Σ and Σ' . The $SU(2)_R$ factor is the ordinary $SU(2)$ R-symmetry in $\mathcal{N} = 2$ SUSY theories that acts on the $\mathcal{Q}^{(4)1}$ and

¹¹Otherwise, we have massless $U(3)_H$ adjoint chiral multiplets, which are charged under the $SU(3)_C$ but not under the $SU(2)_L \times U(1)_Y$, ruining the approximate unification of the standard-model gauge coupling constants.

$\mathcal{Q}^{(4)4}$. This symmetry is broken down to a $U(1)$ symmetry by the FI F-term (see eq.(1)). This $U(1)$ symmetry mixes scalars of chiral multiplet Q_α^k (\bar{Q}_k^α) and scalars of anti-chiral multiplets $\bar{Q}_k^{\alpha\dagger}$ ($Q_\alpha^{k\dagger}$), respectively, and therefore, this symmetry has nothing to do with the usual low-energy R-symmetry. Thus, we regard the $U(1)_R$ in eq.(29) as the origin of the \mathbf{Z}_4 R-symmetry discussed in the previous section. Indeed, R-charges are determined from the symmetry algebra: the $U(3)_H$ adjoint chiral multiplets X_α^β and X_0 have the $U(1)_R$ -charges 2, $(Q^i\bar{Q}_i)$ and $(Q^6\bar{Q}_6)$ carry the $U(1)_R$ -charges 0.¹² This charge assignment is the same as that given in Table I for the $U(3)_H$ -sector particles.¹³ We do not find any reason that the $U(1)_R$ should be broken down to its discrete \mathbf{Z}_{4R} subgroup at this level. However, it becomes clear in section V that this $U(1)_R$ is in fact broken down to a discrete \mathbf{Z}_{4R} in a natural extension of our 6-dimensional brane-world model to a 10-dimensional string theory.

The final comment in this section is that the hypercolor $U(3)_H$ 3-brane should be very close to our 3-brane at an orbifold fixed point. This is because the nonrenormalizable operators in eq.(6) are exponentially suppressed, otherwise. However, as discussed in section II, these operators are necessary to produce the realistic mass spectra for quarks and leptons. In this paper we assume the distance \tilde{D} between the hypercolor $U(3)_H$ brane and our brane to be $\tilde{D} \lesssim 1/M_*$ so that the exponential factor $\exp(-M_*\tilde{D})$ is of $\mathcal{O}(1)$. This arrangement of closely separated two 3-branes is also crucial to have an unsuppressed superpotential eq.(4) between the Higgs and the hyperquark chiral multiplets.

IV. GAUGINO MEDIATION OF THE SUSY BREAKING

We discuss, in this section, the mediation of SUSY breaking that is supposed to occur on a 3-brane at the fixed point F_h of our orbifold $\mathbf{T}^2/\mathbf{Z}_4$ (see Fig.1). We show that the present model is regarded as an extension of the gaugino-mediation model of the SUSY breaking [10,9].

In supergravity, the hidden sector responsible for the SUSY breaking is assumed to be fully separated from the visible sector in order to suppress the unwanted FCNC's [22]. This

¹² Embedding of the $U(1)_R$ into the $SU(4)_R$ determines also the $U(1)_R$ charges of the $SU(5)_{GUT}$ adjoint chiral multiplets: Σ'' has the charge 2, while Σ and Σ' have the charges 0.

¹³ This coincidence of the charge assignment is not accidental, but rather a consequence of the $\mathcal{N} = 2$ SUSY of the theory.

separation is beautifully realized in the brane world as pointed out in ref. [8]. We put the SUSY-breaking sector on a hidden 3-brane at the orbifold fixed point separated from the fixed-point 3-brane on which our visible sector resides. Then, it is obvious to realize the separation of the hidden and visible sectors, if the distance D between the two 3-branes at the orbifold fixed points is sufficiently large and interactions between the two sectors are exponentially suppressed [8]. It seems also natural to postulate that the separation takes place in the “conformal” frame in supergravity [8,23]. If it is the case, the Kähler potential \mathcal{K} and superpotential W have the following forms [9]:

$$\mathcal{L} = \int d^2\Theta \, 2\mathcal{E} \left[-\frac{1}{8}(\bar{\mathcal{D}}\mathcal{D} - 8R) \mathcal{K}(\Phi, \Phi^\dagger, Z, Z^\dagger) + W(\Phi, Z) \right] + \text{h.c.}; \quad (30)$$

$$\mathcal{K}(\Phi, \Phi^\dagger, Z, Z^\dagger) = -3 + f_{\text{vis}}(\Phi, \Phi^\dagger) + f_{\text{hidd}}(Z, Z^\dagger), \quad (31)$$

$$W(\Phi, Z) = W_{\text{vis}}(\Phi) + W_{\text{hidd}}(Z). \quad (32)$$

Here, Φ and Z denote fields in the visible and hidden sectors, respectively. We have employed the superspace notation given in [24]. In the Einstein frame, we see that the Kähler potential K has the form of no-scale type¹⁴ as

$$K(\Phi, \Phi^\dagger, Z, Z^\dagger) = -3 \log \left(1 - \frac{1}{3}f_{\text{vis}}(\Phi, \Phi^\dagger) - \frac{1}{3}f_{\text{hidd}}(Z, Z^\dagger) \right). \quad (33)$$

It is a crucial observation in ref. [9] that all soft SUSY-breaking masses for sfermions and A terms in the visible sector vanish in the limit of the zero cosmological constant.

In the present model the $\text{SU}(5)_{\text{GUT}}$ gauge multiplet lives in the 6 dimensional bulk and can couple directly to the hidden-sector field Z which gives rise to a SUSY-breaking gaugino mass $m_{\tilde{G}_5}$. Thus, the bulk $\text{SU}(5)_{\text{GUT}}$ gaugino \tilde{G}_5 may transmit the SUSY breaking on the hidden 3-brane to the SUSY standard-model sector on our visible 3-brane (*i.e.* gaugino mediation) [10,9].

Let us denote the field responsible for the SUSY breaking Z_0 resides on a hidden 3-brane separated from our visible 3-brane by extra dimensions. The $\text{SU}(5)_{\text{GUT}}$ gaugino acquires the SUSY-breaking mass $m_{\tilde{G}_5}$ through the following interaction with Z_0 ;

$$\mathcal{S} = \int \int \int d^2\theta \sqrt{-g^{(6)}} \eta \frac{Z_0(x, y)}{M_*} \mathcal{W}_\alpha \mathcal{W}^\alpha \delta(y_1 - \frac{1}{2}L) \delta(y_2 - \frac{1}{2}L) d^4x d^2y, \quad (34)$$

¹⁴ The no-scale supergravity [25] adopts a specific form $f_{\text{hidd}} = Z + Z^\dagger$.

where we have taken the position of the hidden brane to be $(y_1 = L/2, y_2 = L/2)$ (an orbifold fixed point). Our visible brane is located at the origin of the extra space $(y_1 = 0, y_2 = 0)$ (see Fig.1). The coupling η in the above equation is a dimensionless constant. From this interaction we obtain the gaugino mass as

$$\frac{m_{\tilde{G}_5}}{g_{\text{GUT}}^2} \simeq \eta \frac{\langle F_{Z_0} \rangle}{M_*}, \quad (35)$$

where $\langle F_{Z_0} \rangle$ is the SUSY-breaking F -term of the field Z_0 . The gravitino mass $m_{3/2}$ is determined as

$$m_{3/2} \simeq \frac{\langle F_{Z_0} \rangle}{M_{\text{Pl}}} \quad (36)$$

and hence we get

$$m_{\tilde{G}_5} \simeq g_{\text{GUT}}^2 \eta \frac{M_{\text{Pl}}}{M_*} m_{3/2}. \quad (37)$$

Since $M_{\text{Pl}}/M_* \simeq \mathcal{O}(10)$, we have relatively large gaugino mass compared to the gravitino mass if $\eta \simeq \mathcal{O}(1)$. We take $\eta \simeq 0.1$ to give the gaugino mass comparable to the gravitino mass. This is because the SUSY-invariant mass μ for the Higgs chiral multiplets ($W = \mu H \bar{H}$) becomes of $\mathcal{O}(m_{3/2})$ as seen below and phenomenological analyses in refs. [9,10] suggest $m_{\tilde{G}_5} \simeq \mu$.

On the other hand, the $U(3)_{\text{H}}$ gauginos remain almost massless, since the $U(3)_{\text{H}}$ 3-brane is separated from the SUSY-breaking hidden 3-brane by extra dimensions, and the $U(3)_{\text{H}}$ vector multiplets do not have a direct coupling to the hidden-sector field Z_0 . We neglect their masses in the present analysis.

Remarkable is that the original $SU(5)_{\text{GUT}} \times U(3)_{\text{H}}$ model predicts the gaugino masses as

$$\frac{m_{\tilde{G}_3}}{\alpha_3} : \frac{m_{\tilde{G}_2}}{\alpha_2} : \frac{m_{\tilde{G}_1}}{\alpha_1} \simeq \left(\frac{m_{\tilde{G}_5}}{g_{\text{GUT}}^2} + \frac{m_{\tilde{H}_3}}{g_{3\text{H}}^2} \right) : \frac{m_{\tilde{G}_5}}{g_{\text{GUT}}^2} : \left(\frac{m_{\tilde{G}_5}}{g_{\text{GUT}}^2} + \frac{m_{\tilde{H}_1}}{15g_{1\text{H}}^2} \right), \quad (38)$$

where $m_{\tilde{H}_3}$ and $m_{\tilde{H}_1}$ are SUSY-breaking masses of the $SU(3)_{\text{H}}$ and the $U(1)_{\text{H}}$ gauginos, respectively. Thus, we expect, in general, a large deviation from the GUT mass relation for the SUSY standard-model gauginos [11]. However, we find here $m_{\tilde{H}_3} \simeq m_{\tilde{H}_1} \simeq 0$, which leads to the standard GUT relation,

$$m_{\tilde{G}_1} \simeq m_{\tilde{G}_2} \simeq m_{\tilde{G}_3}, \quad (39)$$

at the unification scale M_{GUT} , where $\alpha_3 \simeq \alpha_2 \simeq \alpha_1$.

With this gaugino masses the SUSY-breaking masses for squarks and sleptons are generated from one-loop diagrams of intermediate gauginos as shown in [9,10]. Since the gauge interactions are flavor blind, we have degenerate soft SUSY-breaking masses for each sfermions with the same quantum numbers of the standard-model gauge group, and hence we may suppress the unwanted FCNC's.

We are now at the point to discuss the SUSY-invariant mass of the Higgs chiral multiplets (called μ term). It is also important observation in ref. [9] that the μ term naturally arises from the Kähler potential eq.(31) with

$$f_{\text{vis}}(\Phi, \Phi^\dagger) = HH^\dagger + \bar{H}\bar{H}^\dagger + \tau H\bar{H} + h.c., \quad (40)$$

where τ is a dimensionless constant of $\mathcal{O}(1)$. Notice that this Kähler potential is invariant of the continuous $U(1)_R$ symmetry. After the SUSY breaking, the superpotential W eq.(32) should condense so that the cosmological constant vanishes. Therefore, the \mathbf{Z}_{4R} is broken down to the \mathbf{Z}_2 R-parity. In this circumstance, the above Kähler potential induces the μ term and a SUSY-breaking B term for the Higgs multiplets as

$$\mu \simeq \tau m_{3/2}, \quad B \simeq \mu m_{3/2}. \quad (41)$$

Here, B is defined as

$$\mathcal{L} = Bh\bar{h}, \quad (42)$$

where h and \bar{h} are scalar components of the Higgs chiral multiplets H and \bar{H} , respectively. This is basically equivalent to the Giudice-Masiero mechanism [1].

The detailed phenomenology is given in [9,10,26]. We only stress here that the gaugino-mediation model predicts the lightest SUSY particle to be a bino or a right-handed slepton (a super-partner of right-handed charged lepton) at $\mathcal{O}(100)$ GeV.

There may be contact interactions between fields on our visible 3-brane and on the SUSY-breaking hidden 3-brane, which induce, in principle, the dangerous FCNC's. Such contact terms have the following form;

$$\mathcal{L}_{\text{cont}} \sim \frac{e^{-M_* D}}{M_*^2} \int d^4\theta \Phi \Phi^\dagger Z_0 Z_0^\dagger, \quad (43)$$

which give rise to soft SUSY-breaking masses for squarks and sleptons that may be generally flavour dependent. The distance D between our visible and the hidden 3-brane is $D = L/\sqrt{2}$ with $M_* L \simeq 20$. These must be compared with the operators that are induced by gaugino

loops. We easily see that the exponential suppression factor $e^{-\frac{1}{\sqrt{2}}M_*L}$ is $\mathcal{O}(10^{-6})$ in our model, which is small enough to satisfy all constraints from the experimental upper bounds on FCNC's.

V. STRING-THEORY FRAMEWORKS

We have described, in section III, the semi-simple unification model based on the $SU(5)_{\text{GUT}} \times U(3)_{\text{H}}$ gauge group in terms of a 6-dimensional effective field theory. To explain both the quasi- $\mathcal{N} = 2$ SUSY in the $U(3)_{\text{H}}$ sector and the disparity of the gauge coupling constants between the $SU(5)_{\text{GUT}}$ and the $U(3)_{\text{H}}$, we have introduced two types of branes. From the 6-dimensional point of view, one is the $U(3)_{\text{H}}$ 3-brane and the other is the $SU(5)_{\text{GUT}}$ 5-brane. In this section, we embed furthermore our 6-dimensional model into string theories, since they may provide natural frameworks for the brane world. We see that string theories may indeed accommodate the semi-simple unification model. In particular, it is extremely interesting that the $\mathcal{N} = 4$ SUSY in the 6-dimensional space-time is an automatic consequence of D-brane structure in the 10-dimensional string theories. We also show that the string-theory frameworks provide geometric descriptions for the orbifold group, the discrete low-energy \mathbf{Z}_4 R-symmetry and the FI term.

For this purpose we first extend our 6-dimensional model to a 10-dimensional theory, where we assume the four extra 6789 space dimensions to be compactified smaller than the unification scale M_{GUT}^{-1} . The extension is not unique, since there are many 10-dimensional string-theoretical configurations which give an equivalent 6-dimensional theory after the compactification. In fact, the string duality [27] connects one description to another and those are equivalent in the string theory. Therefore, the requirement of reproducing the 6-dimensional configuration given in section III alone cannot determine how many dimensions and in what directions the branes wrap in the compactified four dimensions. In other words, we can choose convenient ones depending on the purpose, and we do change descriptions place by place in what follows.

The $U(3)_{\text{H}}$ 3-brane that is located in the 6-dimensional bulk is naturally lifted up to D3-branes in the type IIB string theory [28]. In addition to the D3-branes, we need another type of branes to hold the $SU(5)_{\text{GUT}}$ on them. As we mentioned in section III and in the appendix B, the existence of a single type of D-branes breaks half of the SUSY charges. One might consider that the two different types of D-branes introduced above leave 1/4 of the

SUSY, and 8 SUSY charges ($\mathcal{N} = 2$ in 4 dimensions) are left unbroken among the 32 SUSY charges of the type IIB string theory, but this is not correct in general. We use D7-branes along with the D3-branes, because it is well known that Dp - $D(p+4)$ brane system keeps the $\mathcal{N} = 2$ SUSY [29]. We also explicitly explain this later. Five D7-branes stretching in 1234567 space directions with three D3-branes in 123 space directions lead to a $U(5) \times U(3)$ gauge theory [30]. We identify the extra $U(1)$ on the D7-branes with $B - L$ symmetry, and hence it naturally accommodates massive right-handed neutrinos.¹⁵

The field contents of the string-theory D3-D7 brane system are as follows: D7-D7 open strings provide a $U(5)_{\text{GUT}}$ vector multiplet $(\mathcal{W}_\alpha^{U(5)_{\text{GUT}}}, \Sigma, \Sigma', \Sigma'')$ of the 6-dimensional (1,1) SUSY ($\mathcal{Q}_{4+}^{(6)} = (\mathcal{Q}^{(4)3,4}), \mathcal{Q}_{4-}^{(6)} = (\mathcal{Q}^{(4)1,2})$). Bosonic part of the $\mathcal{W}_\alpha^{U(5)_{\text{GUT}}}$ and Σ, Σ' arise from the fluctuations of strings into 01234567 directions and the scalar component of Σ'' from the fluctuations into 89 directions.¹⁶ D3-D3 open strings give a $U(3)_{\text{H}}$ vector multiplet $(\mathcal{W}_\alpha^{U(3)_{\text{H}}}, X'' \equiv (X_\alpha^\beta, X_0))$ and $U(3)_{\text{H}}$ adjoint hypermultiplets (X, X') of 4-dimensional $\mathcal{N} = 2$ SUSY ($\mathcal{Q}^{(4)1}, \mathcal{Q}^{(4)4}$). Gauge boson of the $\mathcal{W}_\alpha^{U(3)_{\text{H}}}$ is from the fluctuations in 0123 directions in the D3-branes, scalars of the X and X' from 4567 directions (transverse to the D3- but tangential to the D7-branes) and the scalar of the X'' from the 89 directions (transverse to both of the D3- and the D7-branes). Finally, D3-D7/D7-D3 open strings provide a $U(3)_{\text{H}} \times U(5)_{\text{GUT}}$ bi-fundamental hypermultiplet $(Q_\alpha^k, \bar{Q}_k^\alpha)$ ($\alpha = 1, 2, 3; k = 1, \dots, 5$) of the $\mathcal{N} = 2$ SUSY on the D3-branes.

All these fields are explicitly described as states of open strings, and therefore, the quantum numbers of these fields under the space-time symmetry transformations are unambiguously determined. For later reference, we present their charges under the rotations on the x_4 - x_5 , x_6 - x_7 and x_8 - x_9 planes, respectively, in Table II.

The description of the brane-world model in the above D3-D7 brane system gives an intuitive explanation of the interaction in eq.(1),

$$W = \lambda \bar{Q}_k X'' Q^k. \quad (44)$$

Namely, a displacement of the D3-branes away from the D7-branes (*i.e.* non-zero expectation value of the scalar of X'') leads to massive open strings (Q^k, \bar{Q}_k) stretching between the D3-

¹⁵ The $U(5)_{\text{GUT}} \times U(3)_{\text{H}}$ is broken down to $SU(3)_{\text{C}} \times SU(2)_{\text{L}} \times U(1)_{\text{Y}} \times U(1)_{\text{B-L}}$ by the condensation in eq.(2).

¹⁶ Here, Σ, Σ' and Σ'' represent adjoint multiplets of $U(5)_{\text{GUT}}$, which consist of $\mathbf{24} + \mathbf{1}$ of $SU(5)_{\text{GUT}}$.

and D7-branes.

Next, let us discuss SUSY and R-symmetries. We take the T-duality of the D3-D7 brane system along x_8 and x_9 directions and we discuss in terms of D5-D9 brane system for a while.

We first focus on the $U(5)_{\text{GUT}}$ gauge theory. The D9-brane breaks half of the 32 SUSY charges of the type IIB string theory, and hence $\mathcal{N} = 4$ SUSY is left unbroken in the sense of 4-dimensional effective theory. The condition for unbroken SUSY charges is given as follows. The SUSY charges of the type IIB string theory are represented as two Majorana-Weyl spinors $\mathcal{Q}^{(10)1}$ and $\mathcal{Q}^{(10)2}$ with a common chirality (both $\mathbf{16}_+$ of the $SO(9,1)$). Each of them has 16 real components. We combine them into a complex Weyl spinor $\mathcal{Q}^{(10)} \equiv \mathcal{Q}^{(10)1} + i\mathcal{Q}^{(10)2}$. The condition is written as

$$\mathcal{Q}^{(10)} = \Gamma_{d=10}^{0123456789} \mathcal{Q}^{(10)c}, \quad (45)$$

where the $\mathcal{Q}^{(10)c}$ in the right-hand side is the charge conjugation of the $\mathcal{Q}^{(10)}$, $\Gamma_{d=10}^\mu$ gamma matrices of the $SO(9,1)$, and $\Gamma^{01\cdots 9} \equiv \Gamma^0 \Gamma^1 \cdots \Gamma^9$. (See appendix A for other notations.)

It is known that the type IIB string theory compactified into the 6 dimensions has maximally $SL(5, \mathbf{R})$ symmetry [31]. This symmetry is generated by rotations in the compactified four dimensions and other internal transformations.¹⁷ If we introduce $U(5)_{\text{GUT}}$ D9-branes, the above internal symmetry is broken by the brane-bulk interaction term in the D-brane action and the $SL(5, \mathbf{R})$ symmetry reduces to purely a rotational symmetry of the internal space, that is, to the $SO(4)_{6789}$. The SUSY charges transform as spinors of the $SO(4)_{6789}$, since they form a spinor of the $SO(9,1)$. Therefore, the $SO(4)_{6789}$ is regarded as an R-symmetry, just as the $SO(2)_{45}$ is regarded as an R-symmetry after the dimensional reduction to 4 dimensions as shown in section III. The decomposition of SUSY charge $\mathbf{16}_+$ of the $SO(9,1)$ into the representations of a subgroup $SO(5,1) \times SO(4)_{6789}$ is

$$\mathbf{16}_+ \rightarrow (\mathbf{4}_+, (\mathbf{2}, \mathbf{1})) + (\mathbf{4}_-, (\mathbf{1}, \mathbf{2})). \quad (46)$$

Each term in the right-hand side corresponds to the SUSY charges $Q_{\mathbf{4}_+, A}^{(6)}$ and $Q_{\mathbf{4}_-, B}^{(6)}$, respectively. The rotational symmetry $SO(4)_{6789}$ of the internal space is regarded as the

¹⁷ Precisely speaking, this symmetry is broken down to a discrete subgroup $SL(5, \mathbf{Z})$ due to finite-size effects of the compactification. The spectrum of the Kaluza-Klein modes on the \mathbf{T}^4 torus depends on the shape of the torus and it does not respect the rotational symmetry. This fact plays an important role in our model and we will return to this point. However, we discuss here a symmetry for massless modes and neglect the symmetry breaking.

$SU(2)_{4+} \times SU(2)_{4-}$ R-symmetry of the (1,1) SUSY in the 6-dimension discussed in section III. Due to the reality condition eq.(45), these SUSY charges are pseudo-Majorana-Weyl (see appendix A).

The $U(5)_{\text{GUT}}$ gauge theory on the D9-branes has $\mathcal{N} = 4$ SUSY in 4-dimensional effective theory as stated before. In this description, the rotational symmetry $SO(6)_{456789}$ of the internal 6-dimensional space is identified with the R-symmetry $SU(4)_R$ of the $\mathcal{N} = 4$ SUSY theory discussed in section III. First, we show that the action of the $SO(6)_{456789}$ on the $\mathcal{N} = 4$ SUSY charges, gauginos, and scalars are the same as those of the $SU(4)_R$ given in section III. $(\mathbf{16}_+)$ -spinor (to which the SUSY-charge $\mathcal{Q}^{(10)}$ belongs) and $(\mathbf{16}_-)$ -spinor (to which the $U(5)_{\text{GUT}}$ adjoint gauge fermions and the SUSY-transformation parameters belong) are decomposed as

$$\mathbf{16}_+ \rightarrow (\mathbf{2}_L, \mathbf{4}) + (\mathbf{2}_R, \mathbf{4}^*), \quad (47)$$

$$\mathbf{16}_- \rightarrow (\mathbf{2}_L, \mathbf{4}^*) + (\mathbf{2}_R, \mathbf{4}), \quad (48)$$

of the $SO(3,1) \times SO(6)_{456789}$. Two terms in the right-hand side of eq.(47) correspond to $\mathcal{Q}_\alpha^{(4)a}$ and $\bar{\mathcal{Q}}_a^{(4)\dot{\alpha}}$, respectively.¹⁸ Thus, we have $\mathcal{N} = 4$ SUSY charges belonging to $\mathbf{4}$ -spinor of the $SO(6)_{456789}$ (*i.e.* $\mathbf{4}$ of the $SU(4)_R$). It is easy to see here that the $U(5)_{\text{GUT}}$ adjoint gauge fermions χ_a belong to $\mathbf{4}^*$ -spinor of the $SO(6)_{456789}$, and the $U(5)_{\text{GUT}}$ adjoint scalars $\sigma, \sigma', \sigma''$ to vector of the $SO(6)_{456789}$ (by definition) as in eq.(25). Therefore, the $SO(6)_{456789}$ is really the $SU(4)_R$ symmetry considered in section III. Furthermore, we explicitly find, from the Table II which shows the charges of fields for the maximal torus $SO(2)_{45} \times SO(2)_{67} \times SO(2)_{89}$ of this $SO(6)_{456789}$, that the charges of the 2nd-lowest component fields are smaller than those of the lowest components by the charge of the SUSY generator $\mathcal{Q}^{(4)1}$. Therefore, each of these $SO(2)$ symmetries can be identified with the usual R-symmetry in $\mathcal{N} = 1$ SUSY theories.

Now, let us consider the SUSY breaking $\mathcal{N} = 4 \rightarrow \mathcal{N} = 2$. The SUSY breaking due to the presence of the $U(3)_H$ D5-branes along 012389 directions is represented by the following constraint on the SUSY charges:

$$\mathcal{Q}^{(10)} = -\Gamma_{d=10}^{012389} \mathcal{Q}^{(10)c}. \quad (49)$$

Combining eq.(49) and eq.(45), we obtain a holomorphic constraint on the $\mathcal{Q}^{(10)}$:

¹⁸The reality condition eq.(45) implies that $\mathcal{Q}_\alpha^{(4)a}$'s and $\bar{\mathcal{Q}}_{\dot{\alpha},a}^{(4)}$'s are complex conjugate of each other.

$$\mathcal{Q}^{(10)} = -\Gamma_{d=10}^{4567} \mathcal{Q}^{(10)}. \quad (50)$$

With the convention of gamma matrices described in the appendix A, where generators of the Cartan of the $\text{SO}(6)_{456789} \simeq \text{SU}(4)_R$ are given by¹⁹

$$\begin{aligned} T_{(6)}^{45}|_4 &\equiv \frac{i}{2} \gamma_{(6)}^{45}|_4 = \frac{1}{2} \text{diag}(-1, -1, +1, +1), \\ T_{(6)}^{67}|_4 &\equiv \frac{i}{2} \gamma_{(6)}^{67}|_4 = \frac{1}{2} \text{diag}(-1, +1, -1, +1), \\ T_{(6)}^{89}|_4 &\equiv \frac{i}{2} \gamma_{(6)}^{89}|_4 = \frac{1}{2} \text{diag}(-1, +1, +1, -1), \end{aligned} \quad (51)$$

the condition eq.(50) is rewritten as

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \mathcal{Q}^{(4)1} \\ \mathcal{Q}^{(4)2} \\ \mathcal{Q}^{(4)3} \\ \mathcal{Q}^{(4)4} \end{pmatrix} = \begin{pmatrix} \mathcal{Q}^{(4)1} \\ \mathcal{Q}^{(4)2} \\ \mathcal{Q}^{(4)3} \\ \mathcal{Q}^{(4)4} \end{pmatrix}. \quad (52)$$

From this equation, we see that $\mathcal{Q}^{(4)2}$ and $\mathcal{Q}^{(4)3}$ are broken and the $\mathcal{N} = 2$ SUSY generated by $\mathcal{Q}^{(4)1}$ and $\mathcal{Q}^{(4)4}$ is realized on the D5-branes, as stated before.

Because the D5-branes are wrapped only along two of six internal directions (*i.e.* x_8 and x_9 directions), the $\text{SO}(6)_{456789}$ symmetry is actually broken into $\text{SO}(4)_{4567} \times \text{SO}(2)_{89}$, which is the same as the $\text{SU}(2)_R \times \text{SU}(2)_F \times \text{U}(1)_R$ in eq.(29). What is important here is that the R-symmetries relevant to the phenomenology (*i.e.* $\text{SU}(2)_R$ and $\text{U}(1)_R$) have geometrical interpretations. The $\text{U}(1)_R$ symmetry is the space-rotational symmetry $\text{SO}(2)_{89}$ in the compactified internal x_8 - x_9 plane. The $\text{SU}(2)_R$ and $\text{SU}(2)_F$ are $\text{SU}(2)$ rotation symmetries of complex coordinates $(x_4 + ix_5, x_6 - ix_7)$ and $(x_4 + ix_5, x_6 + ix_7)$, respectively. This can be understood by observing how the $\text{SU}(2)_R$ and $\text{SU}(2)_F$ act on the $\sigma, \sigma', \sigma^*$ and σ'^* (see eq.(19) and eq.(29)).

Finally, we reinterpret the \mathbf{Z}_4 orbifolding of \mathbf{T}^2 in the 6-dimension as one of \mathbf{T}^6 . In the 6 dimensions, the \mathbf{Z}_4 generator in eq.(22) is a $-\pi/2$ rotation of the x_4 - x_5 plane accompanied by an R-rotation. Because an R-rotation is a rotation on the internal space in the string framework and the $\text{SU}(4)_R \simeq \text{SO}(6)_{456789}$ is nothing but the rotation in 456789 directions, we can consider the orbifolding in a totally geometrical way. We find that the eq.(22) is identified with the following rotation:

¹⁹ The $T_{(6)}^{mn}$ is the generator of rotation on x_m - x_n plane.

$$(x_4 + ix_5) \rightarrow e^{-\pi i/2}(x_4 + ix_5), \quad (x_6 + ix_7) \rightarrow e^{-\pi i/2}(x_6 + ix_7), \quad (x_8 + ix_9) \rightarrow e^{\pi i}(x_8 + ix_9). \quad (53)$$

Indeed, with the convention of the γ -matrices in eq.(51), the 4-spinor representation matrix of the rotation eq.(53) is given by

$$\exp\left(\frac{-\pi i}{2}T_{(6)}^{45}\right)\exp\left(\frac{-\pi i}{2}T_{(6)}^{67}\right)\exp\left(\pi iT_{(6)}^{89}\right)|_4 = \text{diag}(1, i, i, -1), \quad (54)$$

and this is identical with the transformation eq.(22). Therefore, the invariance under this orbifold transformation allows only one SUSY charge $\mathcal{Q}^{(4)1}$ to be left unbroken as we have shown in section III.

We obtain a fully geometrical picture of the orbifolding in the 10-dimensional description. The geometry of the compactified manifold in the extra 6 dimensions must be consistent with the orbifold-group symmetry. Two \mathbf{T}^2 's along x_4 - x_5 and x_6 - x_7 directions should be square because the orbifold-group action rotates both x_4 - x_5 and x_6 - x_7 by $-\pi/2$. On the contrary, the torus along x_8 - x_9 directions can be of generic shape, because any toroidal compactification preserve \mathbf{Z}_2 symmetry generated by the angle- π rotation. Taking account that the $\text{SO}(2)_{89}$ also has spinor representations, we see that the unbroken symmetry of the torus in x_8 - x_9 direction is really a \mathbf{Z}_4 . This is identified with the \mathbf{Z}_{4R} symmetry.²⁰ Comparing Table II with Table I, we find Q^k , \bar{Q}_k and $X''(\equiv (X_\beta^\alpha, X_0))$ have desirable charges under this symmetry. Notice that the \mathbf{Z}_4 R-symmetry is a natural consequence of a theory given by a toroidal compactification in a higher dimensional theory, and the 4-dimensional effective theory discussed in section II really has this \mathbf{Z}_4 R-symmetry. It is extremely encouraging that the discreteness of the R-symmetry (and more precisely, the reason why the R-symmetry is broken down to \mathbf{Z}_4) is explained naturally when we extend the 6-dimensional effective theory into this 10-dimensional theory with toroidal compactification.

There is another important feature of our model; that is, the FI F-term parameter V^2 in eq.(1). It is possible to put the FI F-term by hand in the $\text{U}(3)_H$ -brane superpotential, if we regard this 10-dimensional theory just as a field theory. In string theories, FI parameters in the $\mathcal{N} = 2$ SUSY Dp-D(p+4)-brane system may be explained by a string B -field background [32] as we see below.

The 4-dimensional $\mathcal{N} = 2$ SUSY-transformation law of gauginos is given by

²⁰ This \mathbf{Z}_4 R-symmetry is a gauge symmetry originating from the $\text{SO}(9, 1)$.

$$\delta\psi \sim (F_{ij}\gamma_{d=4}^{ij} + \xi_a\tau_a)\epsilon + (\text{terms containing scalar fields}), \quad (55)$$

where the gauginos ψ and the transformation parameter ϵ are $SU(2)_R$ doublets, τ_a the Pauli matrices which act on $SU(2)_R$ doublets, and ξ_a ($a = 1, 2, 3$) are $SU(2)_R$ triplet FI parameters that break the $SU(2)_R$ symmetry. ξ_3 is FI D-term parameter, and $\xi_1 + i\xi_2$ the FI F-term parameter that corresponds to the V^2 in the superpotential eq.(1). (V is of order of $M_{\text{GUT}} \simeq \tilde{L}^{-1}$.) On the other hand, the $\mathcal{N} = 2$ SUSY transformation of gauginos on D-branes is given by

$$\delta\psi^A = \frac{-1}{2}\mathcal{F}_{ij}\gamma_{d=4}^{ij}\epsilon_B\epsilon^{AB} + \frac{-1}{2}\mathcal{F}_{mn}\Gamma^{mn}\epsilon_B\epsilon^{AB} - i\epsilon^{AB}\begin{pmatrix} D & -\sqrt{2}iF^* \\ \sqrt{2}iF & -D \end{pmatrix}_B^C \epsilon_C + \dots, \quad (56)$$

where $\epsilon^{AB} = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}$, $\mathcal{F}_{\mu\nu} = F_{\mu\nu} + B_{\mu\nu}$ ($B_{\mu\nu}$ the string B -field) and $(-\sqrt{2}\text{Im } F, \sqrt{2}\text{Re } F, D)$ are the $SU(2)_R$ triplet auxiliary fields in the $\mathcal{N} = 2$ vector multiplet. Notice that, in the *full* string theory on D-branes, a gauge field $F_{\mu\nu}$ and the B -field always appear in a combination $F_{\mu\nu} + B_{\mu\nu}$. If the B -field has background values, the 2nd term in eq.(56) gives rise to a non-linear shift SUSY transformation, which is nothing but the 2nd term in eq.(55) (*i.e.* the FI contributions). Therefore, it is possible that the FI-term is induced by the B -field background in the extra space dimensions. The B -field has six components in four perpendicular directions to the D5-branes. They are decomposed into anti-self-dual part ($SU(2)_F$ triplet) $(B_{56} - B_{47}, B_{64} - B_{67}, B_{45} - B_{67})$ and self-dual part ($SU(2)_R$ triplet) $(B_{56} + B_{47}, B_{64} + B_{57}, B_{45} + B_{67})$. The latter triplet is identified with the FI parameters ξ_a . To determine the correspondence between three FI-parameters and three self-dual components of the B -field, we examine their behavior under the $U(1)$ R-symmetry generated by $T_{(6)}^{45} + T_{(6)}^{67}$, which is the diagonal $U(1)$ subgroup of the $SU(2)_R$. Among the $SU(2)_R$ triplet $(B_{56} + B_{47}, B_{64} + B_{57}, B_{45} + B_{67})$, only one component $B_{45} + B_{67}$ is invariant under this $U(1)$. Thus, we can conclude that $B_{45} + B_{67}$ is identified with the D-term FI parameter ξ_3 and other two components correspond to the F-term FI parameter $\xi_1 + i\xi_2$.²¹ If the latter components take appropriate expectation values, we can obtain nonzero value of parameter V in the superpotential in eq.(1).

²¹Another derivation of this explicit correspondence between the B -field vev's and the FI parameters is given in the appendix C, where $\mathcal{F}_{mn}\Gamma^{mn}$ is explicitly calculated using our gamma-matrix convention.

Up to now, we have seen that the SUSY, R-symmetry, and some part of the field contents in the 6-dimensional brane world scenario are naturally reproduced in the string framework. If we proceed further, however, we come up against some problems.

The first one is about the Ramond-Ramond (RR) charge cancellation of the branes. For D5-branes, all the four transverse directions are compactified. In such a situation, the total charge of D5-branes should vanish because the flux has nowhere to go. This is also the case for D9-branes. D9-branes fill up the whole 10-dimensional space-time and the flux of the D9-brane charge has really nowhere to go. This severely restricts the number of branes, as we see below.

First, let us discuss the cancellation of the RR charges of the D9-branes. Because each of the D9-branes has the same charge, we cannot put any D9-brane unless we put other kind of objects with opposite sign of the RR charge. For example, we can use an orientifold 9-plane (O9-plane) to cancel the charge of D9-branes without breaking SUSY [28,33]. An orientifold p -plane (Op -plane) is a fixed point of a specific \mathbf{Z}_2 -orbifolding of a torus that flips $(9 - p)$ space directions. In the case of an O9-plane, the \mathbf{Z}_2 -flip is an internal operation reversing the orientation of strings, and this “fixed O9-plane” fills up the whole space-time as the D9-branes do. The charge of an O9-plane is -32 [28,33], if we normalize the RR charge of a D9-brane to be unity. Therefore, if we use an O9-plane to cancel the D9-brane charges, we should put 32 D9-branes in a torus. The type IIB string theory with an O9-plane and 32 D9-branes (compensating the RR charge of the O9-plane) is nothing but the type I string theory. This theory has an $\text{SO}(32)$ gauge group in the 10-dimensional bulk.

This $\text{SO}(32)$ gauge group is much larger than what we want (*i.e.* $\text{U}(5)_{\text{GUT}}$), and hence this symmetry must be somehow broken down. We always have adjoint Higgs fields obtained from the $\text{SO}(32)$ gauge field by the compactification. If these Higgs fields acquire appropriate vev’s, the gauge group would be broken down to a smaller group. However, this mechanism does not work straightforward in our model. This is because the massless modes of the adjoint Higgs fields are eliminated by the orbifolding.²² Thus, space-independent vev’s

²² The orbifolding here is $\mathbf{Z}_4 \times \mathbf{Z}_2 \times \mathbf{Z}_2$, where the last two (“ \mathbf{Z}_2 ”)’s are the space reflections associated with the Op -plane and $O(p+4)$ -plane in the Dp - $D(p+4)$ brane system. In the D5-D9 description, the last “ \mathbf{Z}_2 ” is associated with the O9-plane and does not correspond to an action on the real space. The “orbifold group” is now larger, and it seems that the unbroken $\mathcal{N} = 1$ SUSY discussed in section III is lost. However, the unbroken SUSY condition from the “ \mathbf{Z}_2 ” orbifolding associated with the Op -planes are always equivalent to the condition form Dp -branes, and hence

of these Higgs fields are forbidden. Then, how about the condensation of the Kaluza-Klein modes? There are many Kaluza-Klein modes which are not projected out by the orbifolding. Although condensation of massive Kaluza-Klein modes is impossible within small perturbations from the $\text{SO}(32)$ -symmetric vacuum, scalar fields which are massive at symmetric vacuum generically have other degenerate vacua (where the gauge symmetry is broken) distant from the symmetric vacuum in $\mathcal{N} = 1$ SUSY field theories.²³ Therefore, we cannot reject the possibility that some unknown non-perturbative effects deform the potential of the Kaluza-Klein modes and as a result, the Higgs fields may obtain space-dependent vev's, which break the gauge symmetry $\text{SO}(32)$ down to the desired one.

The charge cancellation for D5-branes also requires the existence of orientifold 5-planes (O5-planes) and a large gauge group. We should break the gauge symmetry on the D5-branes again. It is much easier to obtain the desired gauge group than in the D9 case. This is because the Higgs fields on the D5-branes localize on the D5-branes which can be free from the orbifold fixed points, whereas the D9-branes always include fixed points on them. Thus, we can use the constant vev's of the adjoint Higgs fields to break the gauge symmetry on the D5-branes.

However, the RR-charge cancellation by the O5-planes does not work in the present model. Since we put in the bulk three D5-branes on which the $\text{U}(3)_{\text{H}}$ sector resides, there exist 15 images under the $\mathbf{Z}_4 \times \mathbf{Z}_2 \times \mathbf{Z}_2$, and hence we have 48 D5-branes. This problem can be, however, easily solved by replacing the orbifold group \mathbf{Z}_4 in eq.(22) by a \mathbf{Z}_2 whose generator is now given by $\text{diag}(1, -1, 1, -1)$. Owing to the orientifold reflection " \mathbf{Z}_2 " \times " \mathbf{Z}_2 " essential features remain unchanged.

The condensation of Kaluza-Klein modes (non-constant modes) seems to cause the breakdown of the SUSY. However, field-independent contribution to the SUSY transformation from the derivatives (Kaluza-Klein momenta) of these condensations can be canceled by those from derivative terms of other background fields (such as the metric, B -field, which we discuss later, gauge fields and scalars). Indeed, once the Kaluza-Klein condensation takes place, the uniformity and flatness are lost in the background space-time of the compactified

there occurs no further breaking of the SUSY. We briefly explain this in the appendix D.

²³ For example, an $\mathcal{N} = 1$ SUSY theory described by a superpotential $W = m\Phi^2 + \lambda\Phi^3$ satisfies the property discussed in the text. Notice that the SUSY governing interactions of the $\text{U}(5)_{\text{GUT}}$ multiplets is not $\mathcal{N} = 4$ but $\mathcal{N} = 1$.

manifold. Background metric with nonzero derivative is rather natural. Although this picture is far beyond the small perturbation from the flat-torus-compactification picture, once we accept it, we may be able to resolve other problems of the D-brane construction of our model.

One relevant problem is that we have two unwanted $SU(3)_H$ adjoint chiral multiplets X and X' on the D5-branes. They are NG-bosons (and their fermionic partners) associated with the breaking of the parallel transport symmetry due to the existence of the D5-branes and these bosons represent fluctuations of the D5-branes. To eliminate these massless fields, we have to fix the D5-branes at some point in the compactified manifold. If the background space-time is not uniform, this problem may be solved naturally.²⁴

Furthermore, the condensation of Kaluza-Klein modes might resolve a problem concerning the FI term. We go back to the D3-D7 description again, taking the T-duality.²⁵ Among three components of the B -field corresponding to the three FI parameters ξ_a ($a = 1, 2, 3$), only the D-term parameter $\xi_3 \sim B_{45} + B_{67}$ is invariant under the orbifolding eq.(53). Therefore, if we assume a constant B -field, we cannot obtain nonzero FI F-term parameter $(\xi_1 + i\xi_2) \sim V^2$ in the superpotential eq.(1). However, once the uniformity of the background space-time is broken, the presence of a space-dependent B -field is quite natural. Therefore, there is no reason to forbid the emergence of the nonzero $\xi_1 + i\xi_2$ parameter on the $U(3)_H$ D3-branes and we may expect the desirable value $|\xi_1 + i\xi_2| \sim |V|^2 \sim M_{\text{GUT}}^2$.

The D-brane construction has another kind of difficulty. Unlike the gauge fields and matter fields X'' , Q_α^k and \bar{Q}_k^α , it is not straightforward to accommodate the anti-symmetric tensor $\mathbf{10}$'s of the $SU(5)_{\text{GUT}}$ in the string-framework brane world. However, there have been trials in how to embed the standard model or the anti-symmetric tensor in the string-framework brane world [35–37]. It is possible, for example, that the $\mathbf{10}$'s arise from five D3-branes located at the visible-sector fixed point via an orbifolding whose action on the space-time is accompanied by a rigid gauge transformation. There, the $U(5)_{\text{GUT}}$ is the diagonal subgroup of the fixed-point D3- $U(5)$ and the D7- $U(5)$. We will investigate this possibility in future publication [38]. Similarly, the origin of fields Q_α^6 and \bar{Q}_6^α in the $U(3)_H$

²⁴For example, see [34].

²⁵The T-dual of the orbifold $\mathbf{T}^6/\mathbf{Z}_4$ and orientifold $\mathbf{T}^6/(\mathbf{Z}_2 \times \text{“}\mathbf{Z}_2\text{”} \times \text{“}\mathbf{Z}_2\text{”})$ are $\mathbf{T}^6/\mathbf{Z}_4$ and $\mathbf{T}^6/(\mathbf{Z}_2 \times \text{“}\mathbf{Z}_2\text{”} \times \text{“}\mathbf{Z}_2\text{”})$, where the $\text{“}\mathbf{Z}_2\text{”} \times \text{“}\mathbf{Z}_2\text{”}$ is now the \mathbf{Z}_2 flips associated with the O3- and O7-planes, respectively.

sector is not clear. Because they are neutral with respect to the $SU(5)_{\text{GUT}}$, we cannot simply identify them to D3-D7 (or D5-D9 in the T-dual picture) open strings. One possibility is that they are provided by strings stretching between D3-branes and the orbifold fixed point F_v . Because one of two end points are on the D3-branes, these modes would belong to the fundamental representation of the $SU(3)_H$. To make Q_α^6 and \bar{Q}_6^α light, the D3-branes should be sufficiently close to the fixed point. This seems to induce a side effect that modes of open strings stretching between a D3-brane and its mirror image have the same order of masses as the mass of the Q_α^6 and \bar{Q}_6^α . However, if the background manifold is deformed as we mentioned above, these modes may acquire masses by interactions with the background field and it may be possible to obtain only Q_α^6 and \bar{Q}_6^α .

All arguments above depend on the string dynamics, which seems, however, very unclear at the present stage of understanding the string theories, and it is beyond the scope of this paper. Therefore, we consider that more intense studies on the string dynamics are highly desired for a fully (phenomenologically and theoretically) consistent string-unification model.

VI. DISCUSSION AND CONCLUSIONS

The SUSY $SU(5)_{\text{GUT}} \times U(3)_H$ model [2–4] is quite an attractive candidate for an effective theory at the unification scale $M_{\text{GUT}} \simeq 10^{16}\text{GeV}$, since it naturally explains light Higgs doublets keeping the success of the conventional SUSY grand unified theories. The phenomenological study of the original model suggests a number of mysterious, but interesting features. In particular, the approximate unification of three gauge coupling constants α_3, α_2 and α_1 implies that the hypercolor $U(3)_H$ gauge interactions are in a strong coupling regime. This requirement leads us to propose a relatively lower cut-off scale $M_* \simeq 10^{17}\text{GeV}$, since the abelian $U(1)_H$ gauge coupling constant blows up below the Planck scale $M_{\text{Pl}} \simeq 2 \times 10^{18}\text{GeV}$. Thus, we are forced to embed the original semi-simple $SU(5)_{\text{GUT}} \times U(3)_H$ model in the brane world in a higher dimensional space-time, in which the Planck scale M_{Pl} is merely an effective one and the fundamental scale is rather the cut-off scale M_* .

In this paper, we assume a 6-dimensional space-time and consider that the two extra dimensions are compactified into the orbifold $\mathbf{T}^2/\mathbf{Z}_4$ which has two \mathbf{Z}_4 fixed points. The standard quark, lepton and Higgs chiral multiplets are assumed to reside on a 3-brane at one of the orbifold fixed points. We put the $SU(5)_{\text{GUT}}$ gauge vector multiplet in the 6-dimensional bulk. On the other hand, the hypercolor $U(3)_H$ -sector fields are assumed to reside on a 3-brane in the 6-dimensional bulk. A crucial assumption is the $\mathcal{N} = 4$ SUSY in

the 6-dimensional bulk, which is broken down to $\mathcal{N} = 1$ SUSY by the \mathbf{Z}_4 orbifolding. This $\mathcal{N} = 4$ SUSY is very important to preserve the quasi- $\mathcal{N} = 2$ structure in the $U(3)_H$ sector that is one of the mysteries in the original $SU(5)_{\text{GUT}} \times U(3)_H$ unification model.

The present brane-world scenario is very interesting, since it may explain naturally the various mysterious features in the original model. However, it produces another mystery that is the $\mathcal{N} = 4$ SUSY in the 6-dimensional bulk. The necessity of $\mathcal{N} = 4$ SUSY leads us again to postulate further a higher dimensional space-time, that is a 10-dimensional one. It may be surprising that the phenomenological consideration suggests strongly some 10-dimensional supergravity theory. Clearly, the most attractive candidate for it is the superstring theories. We find, in this paper, that the type IIB string theory with D3-D7 brane structure may accommodate our 6-dimensional brane-world model, where the $\mathcal{N} = 4$ SUSY is the automatic prediction of the theory. The phenomenologically required natures of the $U(3)_H$ 3-brane fields are naturally explained in the type IIB string framework, which seems a miraculous success in this framework, since those natures are just imposed by hand in the original unification model. However, there are many unsolved problems in the present string-theory framework as pointed out in section V. In particular, it is not easy to have the chiral nature $\mathbf{5}^* + \mathbf{10}$ of the quark and lepton multiplets. The RR charge cancellation suggests an $SO(32)$ gauge symmetry on D9-branes (T-dual to the D7-branes) instead of the $U(5)_{\text{GUT}}$ and hence we have to make a dynamical assumption that the $SO(32)$ is broken down to $U(5)_{\text{GUT}} \times (\text{something})$. However, it does not seem obvious to justify such a dynamical breaking at the present knowledge of the string theories.²⁶ Even so, we view the fact that the $\mathcal{N} = 4$ SUSY in the 6-dimensional bulk, the discrete \mathbf{Z}_4 R-symmetry, the FI term and an adequate matter content of the $U(3)_H$ sector can be naturally obtained in the string framework as quite encouraging, and hence consider that the present string-unification model deserves further investigations.

Furthermore, it is very interesting that some of the predictions of the present model are experimentally testable in future. One is the $U(1)$ factor associated with D7-branes, which may be regarded as the $B-L$ symmetry. The presence of the $U(1)$ implies massive neutrinos [39] that are indeed almost confirmed by neutrino oscillation experiments [40]. Another is

²⁶Another way of solving this problem will be given by postulating non-compactified 8-9 directions which may form an anti-de Sitter space. We will discuss this possibility in the future publication [38].

the GUT-like relation of gaugino mass as discussed in section IV, which may be tested in near future collider experiments. As we discussed in section IV the gaugino-mediation model of the SUSY breaking is the most natural in the present brane-world model, which may be also tested in future experiments, since it predicts a very peculiar spectrum for many SUSY particles [9,10,26]

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APPENDIX A: CONVENTIONS OF GAMMA MATRICES AND SPINORS

1. The $SO(3,1)$ gamma matrices and spinors

The metric convention we take in this paper is $\eta_{\mu\nu} = \text{diag}(-1, +1, \dots, +1)$.

4-dimensional gamma matrices $\gamma_{d=4}^i$ ($i = 0, 1, 2, 3$) are given by 4×4 matrices

$$\gamma_{d=4}^i \equiv \begin{pmatrix} & i\sigma^i \\ i\bar{\sigma}^i & \end{pmatrix} \equiv \left(\begin{pmatrix} & i\mathbf{1} \\ i\mathbf{1} & \end{pmatrix}, \begin{pmatrix} & i\vec{\tau} \\ -i\vec{\tau} & \end{pmatrix} \right), \quad (\text{A1})$$

where $\vec{\tau}$ are the Pauli matrices. Using 4×4 matrices $B_{d=4}$ and $C_{d=4}$

$$B_{d=4} \equiv i\gamma^0\gamma^1\gamma^3, \quad B_{d=4}^{-1}(\gamma^{ij})^* B_{d=4} = \gamma^{ij}, \quad (\text{A2})$$

$$C_{d=4} \equiv \gamma^1\gamma^3, \quad C_{d=4}^{-1}(-\gamma^{ij})^T C_{d=4} = \gamma^{ij}, \quad (\text{A3})$$

we have four 4-component spinors²⁷

$$\Psi \equiv \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}, \quad \Psi^c \equiv B_{d=4}^{-1}\Psi^* = \begin{pmatrix} \chi_\alpha \\ -\bar{\psi}^{\dot{\alpha}} \end{pmatrix}, \quad (\text{A4})$$

$$\bar{\Psi} \equiv \Psi^\dagger(-i\gamma^0) = \left(\chi^\alpha \quad \bar{\psi}_{\dot{\alpha}} \right), \quad \bar{\Psi}_c \equiv \Psi^T C_{d=4}^T = \overline{(\Psi^c)} = \left(-\psi^\alpha \quad \bar{\chi}_{\dot{\alpha}} \right), \quad (\text{A5})$$

which transform as

$$\begin{aligned} \Psi &\longrightarrow g \cdot \Psi, & \Psi^c &\longrightarrow g \cdot \Psi^c, \\ \bar{\Psi} &\longrightarrow \bar{\Psi} \cdot g^{-1}, & \bar{\Psi}_c &\longrightarrow \bar{\Psi}_c \cdot g^{-1}, \end{aligned} \quad (\text{A6})$$

²⁷ c of Ψ^c means that Ψ^c is a charge conjugation of Ψ .

where g is the spinor representation of $\text{SO}(3, 1)$. We define the chirality operator $\gamma_{(4)} \equiv (-i\gamma_{d=4}^{0123}) = \text{diag}(1, 1, -1, -1)$.²⁸ Spinor on which $\gamma_{(4)}$ is +1 is referred to as $(\mathbf{2}, \mathbf{1})$ representation and spinor on which $\gamma_{(4)}$ is -1 as $(\mathbf{1}, \mathbf{2})$ representation. The Majorana condition is given by

$$\Psi = \gamma_{(4)} \Psi^c \quad \left(\text{equivalent to } \Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix} \right). \quad (\text{A7})$$

2. The $\text{SO}(5, 1)$ gamma matrices and spinors

6-dimensional gamma matrices $\Gamma_{d=6}^\mu$ ($\mu = 0, 1, \dots, 5$) are given by

$$\Gamma_{d=6}^i = \gamma_{d=4}^i \otimes \gamma_{(2)} \quad (i = 0, 1, 2, 3), \quad \Gamma_{d=6}^m = \mathbf{1}_{4 \times 4} \otimes \gamma_{(2)}^m \quad (m = 4, 5), \quad (\text{A8})$$

where $\gamma_{(2)}^m \equiv (\tau^2, \tau^1)$ and $\gamma_{(2)} \equiv i\gamma_{(2)}^{45} = \tau^3$. Using $(4 \times 4) \otimes (2 \times 2)$ matrices $B_{d=6}$ and $C_{d=6}$

$$B_{d=6} \equiv B_{d=4} \otimes \tau^1 = -\Gamma_{d=6}^{0134}, \quad B_{d=6}^{-1} (\Gamma^{\mu\nu})^* B_{d=6} = \Gamma^{\mu\nu}, \quad (\text{A9})$$

$$C_{d=6} \equiv C_{d=4} \otimes i\tau^2 = i\Gamma_{d=6}^{134}, \quad C_{d=6}^{-1} (-\Gamma^{\mu\nu})^T C_{d=6} = \Gamma^{\mu\nu}, \quad (\text{A10})$$

we have four 8-component spinors (in 4×2 matrix form)

$$\begin{aligned} \Psi &\equiv \begin{pmatrix} \psi_\alpha & \lambda_\alpha \\ \bar{\chi}^{\dot{\alpha}} & \bar{\kappa}^{\dot{\alpha}} \end{pmatrix}, \quad \Psi^c \equiv B_{d=6}^{-1} \Psi^* = \begin{pmatrix} \kappa & \chi \\ -\bar{\lambda} & -\bar{\psi} \end{pmatrix}, \\ \bar{\Psi} &\equiv \Psi^\dagger (-i\Gamma_{d=6}^0) = \begin{pmatrix} \chi^\alpha & \bar{\psi}^{\dot{\alpha}} \\ -\kappa^\alpha & -\bar{\lambda}^{\dot{\alpha}} \end{pmatrix}, \quad \bar{\Psi}_c \equiv \Psi^T C_{d=6}^T = \overline{(\Psi^c)} = \begin{pmatrix} -\lambda & \bar{\kappa} \\ \psi & -\bar{\chi} \end{pmatrix}, \end{aligned} \quad (\text{A11})$$

which transform the same as in eq.(A6) under the $\text{SO}(5, 1)$. We define the chirality operator $\Gamma_{d=6}^{012345} = (\gamma_{(4)} \otimes \gamma_{(2)})$. The chirality +1 part of a spinor is referred to as $\mathbf{4}_+$ representation (*i.e.* ψ and $\bar{\kappa}$ of Ψ in eq.(A11)), and -1 part as $\mathbf{4}_-$ representation (*i.e.* $\bar{\chi}$ and λ). Since $\mathbf{4}_\pm^* \simeq \mathbf{4}_\pm$, the pseudo-Majorana condition can be imposed separately upon $\mathbf{4}_+$ spinors $\{\Psi_{\mathbf{4}_+, A}\}_{A=1,2,\dots,2N_+}$ or $\mathbf{4}_-$ spinors $\{\Psi_{\mathbf{4}_-, B}\}_{B=1,2,\dots,2N_-}$:

$$\Psi_{\mathbf{4}_+, A} = \mathbf{J}_{AC} \Psi_{\mathbf{4}_+, C}^c, \quad \Psi_{\mathbf{4}_-, B} = \mathbf{J}_{BD} \Psi_{\mathbf{4}_-, D}^c, \quad \text{where } \mathbf{J}_{AC} = \mathbf{J}_{BD} = \begin{pmatrix} & \mathbf{1} \\ -\mathbf{1} & \end{pmatrix}. \quad (\text{A12})$$

Or equivalently,

$$\Psi_{\mathbf{4}_+, A} = \begin{pmatrix} \psi_A & \\ & -\bar{\psi}^{A+N_+} \end{pmatrix}, \quad \Psi_{\mathbf{4}_+, A+N_+} = \begin{pmatrix} \psi_{A+N_+} & \\ & \bar{\psi}^A \end{pmatrix}, \quad (\text{A13})$$

$$\Psi_{\mathbf{4}_-, B} = \begin{pmatrix} & \chi^{B+N_-} \\ \bar{\chi}^B & \end{pmatrix}, \quad \Psi_{\mathbf{4}_-, B+N_-} = \begin{pmatrix} & -\chi^B \\ \bar{\chi}^{B+N_-} & \end{pmatrix}. \quad (\text{A14})$$

²⁸We frequently use the abbreviated notation such as $\gamma^{0123} \equiv \gamma^0 \gamma^1 \gamma^2 \gamma^3$.

3. The SO(9, 1) gamma matrices and spinors

10-dimensional gamma matrices $\Gamma_{d=10}^\mu$ ($\mu = 0, 1, \dots, 9$) are given by

$$\Gamma_{d=10}^i = \gamma_{(4)}^i \otimes \gamma_{(6)} \quad (i = 0, 1, 2, 3), \quad \Gamma_{d=10}^m = \mathbf{1}_{4 \times 4} \otimes \gamma_{(6)}^m \quad (m = 4, \dots, 9), \quad (\text{A15})$$

where

$$\begin{aligned} \gamma_{(6)}^4 &= \tau^2 \otimes \mathbf{1} \otimes \tau^2, & \gamma_{(6)}^6 &= \tau^3 \otimes \tau^2 \otimes \tau^2, & \gamma_{(6)}^8 &= \tau^1 \otimes \tau^2 \otimes \tau^2, \\ \gamma_{(6)}^5 &= -\tau^2 \otimes \tau^3 \otimes \tau^1, & \gamma_{(6)}^7 &= -\mathbf{1} \otimes \tau^2 \otimes \tau^1, & \gamma_{(6)}^9 &= -\tau^2 \otimes \tau^1 \otimes \tau^1, \end{aligned} \quad (\text{A16})$$

and

$$\gamma_{(6)} \equiv i\gamma_{(6)}^{456789} = \mathbf{1} \otimes \mathbf{1} \otimes \tau^3. \quad (\text{A17})$$

Using $(4 \times 4) \otimes (8 \times 8)$ matrices $B_{d=10}$ and $C_{d=10}$

$$B_{d=10} \equiv B_{d=4} \otimes (\mathbf{1} \otimes \mathbf{1} \otimes -i\tau^2) = \Gamma_{d=10}^{013579}, \quad B_{d=10}^{-1} (\Gamma^{\mu\nu})^* B_{d=10} = \Gamma^{\mu\nu}, \quad (\text{A18})$$

$$C_{d=10} \equiv C_{d=4} \otimes (\mathbf{1} \otimes \mathbf{1} \otimes \tau^1) = -i\Gamma_{d=10}^{13579}, \quad C_{d=10}^{-1} (-\Gamma^{\mu\nu})^T C_{d=10} = \Gamma^{\mu\nu}, \quad (\text{A19})$$

we have four 32-component spinors (in 4×8 matrix form)

$$\begin{aligned} \Psi &\equiv \begin{pmatrix} \psi_\alpha^a & \lambda_{\alpha,a} \\ \bar{\chi}^{\dot{\alpha},a} & \bar{\kappa}_{\dot{\alpha}} \end{pmatrix}, & \Psi^c &\equiv B_{d=10}^{-1} \Psi^* = \begin{pmatrix} \kappa^a & -\chi_a \\ -\bar{\lambda}^a & \bar{\psi}_a \end{pmatrix}, \\ \bar{\Psi} &\equiv \Psi^\dagger (-i\Gamma_{d=10}^0) = \begin{pmatrix} \chi_a^\alpha & \bar{\psi}_{\dot{\alpha},a} \\ -\kappa^{\alpha,a} & -\bar{\lambda}_{\dot{\alpha}}^a \end{pmatrix}, & \bar{\Psi}_c &\equiv \Psi^T C_{d=6}^T = \overline{(\Psi^c)} = \begin{pmatrix} -\lambda_a & \bar{\kappa}_a \\ -\psi^a & \bar{\chi}^a \end{pmatrix}, \end{aligned} \quad (\text{A20})$$

($a = 1, 2, 3, 4$) which transform the same as in eq.(A6) under the SO(9, 1). We define the chirality operator $\Gamma_{d=10} \equiv \Gamma_{d=10}^{0123456789} = (\gamma_{(4)} \otimes \gamma_{(6)})$. The chirality +1 part of a spinor is referred to as $\mathbf{16}_+$ representation (*i.e.* ψ^a and $\bar{\kappa}_a$ of Ψ in eq.(A20)), and -1 part as $\mathbf{16}_-$ representation (*i.e.* $\bar{\chi}^a$ and λ_a). Since $\mathbf{16}_\pm^* \simeq \mathbf{16}_\pm$, the Majorana condition can be imposed separately upon $\mathbf{16}_+$ spinor $\Psi_{\mathbf{16}_+}$ or $\mathbf{16}_-$ spinor $\Psi_{\mathbf{16}_-}$:

$$\Psi_{\mathbf{16}_\pm} = \Psi_{\mathbf{16}_\pm}^c, \quad (\text{A21})$$

or equivalently,

$$\Psi_{\mathbf{16}_+} = \begin{pmatrix} \psi_\alpha^a \\ \bar{\chi}^{\dot{\alpha},a} \end{pmatrix}, \quad \Psi_{\mathbf{16}_-} = \begin{pmatrix} -\chi_{\alpha,a} \\ \bar{\psi}_{\dot{\alpha}} \end{pmatrix}. \quad (\text{A22})$$

4. Reduction from $d = 10$ to $d = 6$

Let us consider the action of gamma matrices $\Gamma_{d=10}^{0,1,\dots,5}$ on 32-component spinors.

Since the 2nd factor of $\gamma_{(6)}^{4,5}$ (in eq.(A16)) and $\gamma_{(6)}$ (in eq.(A17)) are all $\mathbf{1}$ or τ^3 , the $\text{SO}(5,1)$ action does not mix

$$\begin{pmatrix} \psi^a & \lambda_a \\ \bar{\chi}^a & \bar{\kappa}_a \end{pmatrix}_{a=1,2} \quad \text{and} \quad \begin{pmatrix} \psi^a & \lambda_a \\ \bar{\chi}^a & \bar{\kappa}_a \end{pmatrix}_{a=3,4}. \quad (\text{A23})$$

On each part ($a = 1, 2$ and $a = 3, 4$),

$$\Gamma_{d=10}^4 = \mathbf{1}_{4 \times 4} \otimes (\tau^2 \otimes \gamma_{(2)}^4), \quad \Gamma_{d=10}^4 = \mathbf{1}_{4 \times 4} \otimes (\tau^2 \otimes \gamma_{(2)}^4), \quad (\text{A24})$$

$$\Gamma_{d=10}^5 = \mathbf{1}_{4 \times 4} \otimes (-\tau^2 \otimes \gamma_{(2)}^5), \quad \Gamma_{d=10}^5 = \mathbf{1}_{4 \times 4} \otimes (\tau^2 \otimes \gamma_{(2)}^5). \quad (\text{A25})$$

If we rearrange the basis of spinors as follows:

$$\Psi_{\mathbf{8},A} = \begin{pmatrix} i\lambda_a & \epsilon_{ae}\psi^e \\ -\bar{\kappa}_a & i\epsilon_{ae}\bar{\chi}^e \end{pmatrix}_{e,a=A=1,2}, \quad \Psi_{\mathbf{8},B} = \begin{pmatrix} -\epsilon_{bd}\psi^d & i\lambda_b \\ i\epsilon_{bd}\bar{\chi}^d & \bar{\kappa}_b \end{pmatrix}_{d,b=B+2=3,4}, \quad (\text{A26})$$

then the gamma matrices are now the same as those given in eq.(A8) on each $\Psi_{\mathbf{8},A}$ ($A = 1, 2$) or $\Psi_{\mathbf{8},B}$ ($B = 1, 2$). We can see that the 10-dimensional Majorana condition in eq.(A21) is equivalent to the 6-dimensional pseudo-Majorana conditions on $\{\Psi_{\mathbf{8},A}\}_{A=1,2}$ and $\{\Psi_{\mathbf{8},B}\}_{B=1,2}$ in eq.(A12).

APPENDIX B: SUSY BREAKING DUE TO THE PRESENCE OF A 3-BRANE

Algebra of 6-dimensional (0,1) SUSY is given by

$$\begin{aligned} \{\mathcal{Q}_{4-,B}^{(6)}, \mathcal{Q}_{4-,D}^{(6)}\} &= -i\epsilon_{BD} \left(\Gamma_{d=6}^\mu C_{d=6}^{T,-1} \right) P_\mu \\ &\quad + \tau_B^a C_{CD} \left(\Gamma_{d=6}^{\lambda\mu\nu} C_{d=6}^{T,-1} \right) C_{\lambda\mu\nu}^a + i\epsilon_{BD} \left(\Gamma_{d=6}^{\mu\nu\rho\sigma\tau} C_{d=6}^{T,-1} \right) C_{\mu\nu\rho\sigma\tau}, \end{aligned} \quad (\text{B1})$$

where the $C_{\lambda\mu\nu}^a$ ($a = 1, 2, 3$) in the 2nd term and $C_{\mu\nu\rho\sigma\tau}$ in the 3rd term in the right hand side are charges of objects that extend in 3 and 5 spatial dimensions, that is, charges of 3-brane and 5-brane (6-dimensional bulk itself), respectively. Let us consider the case where a 3-brane exists and see how the unbroken SUSY charge is determined. Suppose that $\langle C_{123}^3 \rangle \neq 0$. (This corresponds to the assumption that the 3-brane stretches in 123 space directions.²⁹) Reduction of this algebra into that of 4-dimensional effective theory is given by

²⁹ $\tau_B^a C \langle C_{123}^a \rangle \neq 0$ breaks $\text{SU}(2)_{4-}$ R-symmetry. When the D5-branes (in the 10-dimensional picture) wrap into 89 directions, the breaking of the $\text{SO}(4)_{6789} \simeq \text{SU}(2)_{4-} \times \text{SU}(2)_{4+}$ is precisely given by $\langle C_{123}^3 \rangle \neq 0$.

$$\{\mathcal{Q}_\alpha^{(4)2}, \bar{\mathcal{Q}}_{\dot{\beta},2}^{(4)}\} = i(i\sigma^i)_{\alpha\dot{\beta}}P_i - (i\sigma^1 i\bar{\sigma}^2 i\sigma^3)_{\alpha\dot{\beta}}C_{123}^3, \quad (\text{B2})$$

$$\{\mathcal{Q}_\alpha^{(4)1}, \bar{\mathcal{Q}}_{\dot{\beta},1}^{(4)}\} = i(i\sigma^i)_{\alpha\dot{\beta}}P_i + (i\sigma^1 i\bar{\sigma}^2 i\sigma^3)_{\alpha\dot{\beta}}C_{123}^3, \quad (\text{B3})$$

$$\{\mathcal{Q}_\alpha^{(4)1}, \mathcal{Q}_\beta^{(4)2}\} = \epsilon_{\alpha\beta}(-P_4 + iP_5). \quad (\text{B4})$$

Then it is easily seen that $\mathcal{Q}^{(4)1}$ is left unbroken (eq.(B3)) while $\mathcal{Q}^{(4)2}$ not (eq.(B2)), if the 3-brane tension $\langle P^0 \rangle$ is related to the 3-brane charge $\langle C_{123}^3 \rangle$ as $\langle P^0 \rangle = -\langle P_0 \rangle = \langle C_{123}^3 \rangle$ (*i.e.* BPS condition).³⁰ Otherwise, both $\mathcal{Q}^{(4)1}$ and $\mathcal{Q}^{(4)2}$ are broken.

The same argument as above holds also in the 6-dimensional (1,0) SUSY. Therefore, to keep the $\mathcal{N} = 2$ SUSY on the 3-brane, we need (1,1) SUSY or (2,0) SUSY in the 6-dimensional bulk, and each 6-dimensional SUSY charge provides one 4-dimensional SUSY charge, if the 3-brane satisfies the BPS condition.

APPENDIX C: EXTENDED SUSY TRANSFORMATION IN 4-DIMENSIONAL GAUGE THEORIES

When a 4-dimensional effective theory is derived from a 10-dimensional theory through a compactification, the $\text{SO}(6)_{456789}$ subgroup is regarded as an internal symmetry. The (4×8) -matrix expression of the 32-component spinor is now regarded as eight 4-component $\text{SO}(3,1)$ spinors, and these eight spinors form a $\mathbf{4} + \mathbf{4}^*$ multiplet under the internal symmetry $\text{SU}(4)_R$. $\mathbf{4}^*$ representation of the $\text{SU}(4)_R$ is simply given by complex conjugate of $\mathbf{4}$ with the convention of the $\text{SO}(6)_{456789}$ gamma matrices in eq.(A16). Explicit representation of the $\text{SU}(4)_R$ can be calculated from eq.(A16). For example, generators of the Cartan subalgebra are given by

$$\frac{i}{2}\gamma_{(6)}^{45} = \frac{-1}{2}\mathbf{1} \otimes \tau^3 \otimes \tau^3, \quad \frac{i}{2}\gamma_{(6)}^{67} = \frac{-1}{2}\tau^3 \otimes \mathbf{1} \otimes \tau^3, \quad \frac{i}{2}\gamma_{(6)}^{89} = \frac{-1}{2}\tau^3 \otimes \tau^3 \otimes \tau^3. \quad (\text{C1})$$

SUSY charge in 10-dimensional SUSY Yang-Mills theories and its infinitesimal transformation parameter are

$$\mathcal{Q}_{\mathbf{16}_+}^{(10)} = \begin{pmatrix} \mathcal{Q}_\alpha^{(4),a} & \\ & \bar{\mathcal{Q}}_a^{(4)\dot{\alpha}} \end{pmatrix} \quad \epsilon_{\mathbf{16}_-} = \begin{pmatrix} & -\epsilon_{\alpha,a} \\ \bar{\epsilon}^{\dot{\alpha},a} & \end{pmatrix}. \quad (\text{C2})$$

Infinitesimal SUSY transformation is decomposed into 4-dimensional spinors as

³⁰ Although the momenta P_4 and P_5 are not well-defined since the 3-brane breaks these symmetries, they do not appear in the SUSY algebra of the unbroken $\mathcal{Q}^{(4)1}$.

$$\bar{\epsilon}_c \mathcal{Q} = \begin{pmatrix} \epsilon_a \\ \bar{\epsilon}^a \end{pmatrix} \begin{pmatrix} \mathcal{Q}^{(4),a} \\ \bar{\mathcal{Q}}_a^{(4)} \end{pmatrix} \rightarrow \epsilon_a \mathcal{Q}^{(4),a} + \bar{\epsilon}^a \bar{\mathcal{Q}}_a^{(4)}. \quad (\text{C3})$$

Here, $\bar{\epsilon}_c$ in the left-hand side is a 10-dimensional spinor defined as in eq.(A20). The 4-dimensional SUSY charges $\mathcal{Q}^{(4),a}$ transform under the $\text{SU}(4)_R$ as $\mathbf{4}$ and $\bar{\mathcal{Q}}_a^{(4)}$ as $\mathbf{4}^*$. Transformation parameters ϵ_a transform as $\mathbf{4}^*$ and $\bar{\epsilon}^a$ as $\mathbf{4}$. Gauge fermions in the 10-dimensional Yang-Mills theory are described by $\mathbf{16}_-$ spinor,

$$\Lambda_{\mathbf{16}_-} = \begin{pmatrix} & -\chi_{\alpha,a} \\ \bar{\chi}^{\dot{\alpha},a} & \end{pmatrix}, \quad (\text{C4})$$

where χ_a transform under the $\text{SU}(4)_R$ as $\mathbf{4}^*$.

SUSY transformation of the 10-dimensional SUSY Yang-Mills theory is given by

$$\delta A_\mu = \bar{\epsilon}_c \Gamma_{d=10,\mu} \Lambda, \quad (\text{C5})$$

$$\delta \Lambda = \frac{1}{2} \Gamma_{d=10}^{\mu\nu} F_{\mu\nu} \epsilon, \quad (\text{C6})$$

whose decomposition into 4-dimension (*i.e.* 4-dimensional $\mathcal{N} = 4$ SUSY transformation) is

$$\delta A_i = i (\epsilon_a \sigma_i \bar{\chi}^a - \chi_a \sigma_i \bar{\epsilon}^a), \quad (\text{C7})$$

$$\delta \sigma_{ab} = \sqrt{2} \{ (\epsilon_a \chi_b - \epsilon_b \chi_a) - \epsilon_{abcd} \bar{\epsilon}^c \bar{\chi}^d \}, \quad (\text{C8})$$

$$\delta \chi_a = \frac{1}{2} \gamma_{d=4}^{ij} \epsilon_a F_{ij} + \sqrt{2} i \sigma^i \bar{\epsilon}^b (D_i \sigma_{ab}) - i [\sigma_{ab}, \sigma^{\dagger,bc}] \epsilon_c. \quad (\text{C9})$$

Here, ϵ_{abcd} is the totally anti-symmetric tensor, D_i the covariant derivatives, and σ_{ab} is defined as eq.(19), where $\sigma \equiv (1/\sqrt{2})(A_4 + iA_5)$, $\sigma' \equiv (1/\sqrt{2})(A_6 + iA_7)$ and $\sigma'' \equiv (1/\sqrt{2})(A_8 + iA_9)$. The 1st term in the right-hand side of eq.(C9) comes from $\Gamma^{ij} F_{ij}$ of eq.(C6), the 2nd term of eq.(C9) from $\Gamma^{in} F_{in}$ of eq.(C6) and the 3rd term from $\Gamma^{mn} F_{mn}$ ($i, j = 0, 1, 2, 3$, $m, n = 4, \dots, 9$). If there exists a background flux $\langle F_{mn} \rangle \neq 0$, then another contribution arises from the $\Gamma^{mn} F_{mn}$ in eq.(C9). This contribution is given by

$$\delta \Lambda = \frac{1}{2} \Gamma^{mn} \langle F_{mn} \rangle \epsilon; \quad (\text{C10})$$

that is,

$$\delta \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \end{pmatrix} = \frac{i}{2} \begin{pmatrix} -\eta'' - \eta - \eta' & 2\eta_+ & 2\eta'_+ & 2\eta''_+ \\ 2\eta_- & -\bar{\eta}'' + \eta + \bar{\eta}' & 2\bar{\eta}''_+ & 2\bar{\eta}'_- \\ 2\eta'_- & 2\bar{\eta}''_- & \bar{\eta}'' - \bar{\eta} + \eta' & 2\bar{\eta}_+ \\ 2\eta''_- & 2\bar{\eta}'_+ & 2\bar{\eta}_- & \eta'' + \bar{\eta} - \bar{\eta}' \end{pmatrix} \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \\ \epsilon_4 \end{pmatrix}, \quad (\text{C11})$$

where

$$\eta \equiv \eta^3 \quad \eta_{\pm} \equiv \eta^1 \pm i\eta^2 \quad \eta^a \equiv \eta^{a,mn} \langle F_{mn} \rangle \quad (a = 1, 2, 3; m, n = 6789), \quad (\text{C12})$$

$$\eta' \equiv \eta'^3 \quad \eta'_{\pm} \equiv \eta'^1 \pm i\eta'^2 \quad \eta'^a \equiv \eta'^{a,mn} \langle F_{mn} \rangle \quad (a = 1, 2, 3; m, n = 8945), \quad (\text{C13})$$

$$\eta'' \equiv \eta''^3 \quad \eta''_{\pm} \equiv \eta''^1 \pm i\eta''^2 \quad \eta''^a \equiv \eta''^{a,mn} \langle F_{mn} \rangle \quad (a = 1, 2, 3; m, n = 4567). \quad (\text{C14})$$

Here, $\eta^{a,mn}$ is the 'tHooft symbol [41]. $\bar{\eta}, \bar{\eta}', \bar{\eta}'', \bar{\eta}_{\pm}, \bar{\eta}'_{\pm}, \bar{\eta}''_{\pm}$ are also defined in an analogous way, using the 'tHooft symbol $\bar{\eta}^{a,mn}$.

The 4-dimensional $\mathcal{N} = 2$ SUSY transformation (transformation parameter $(\delta\theta)_A$) of fields in an $\mathcal{N} = 2$ vector multiplet (scalar $\phi \equiv i\sigma$, gaugino ψ^A and vector A_i) is given by

$$\delta A_i = i \left((\delta\theta)_A \sigma_i \bar{\psi}_B \epsilon^{AB} - \psi^A \sigma_i \overline{(\delta\theta)}^B \epsilon_{AB} \right) \quad (\text{C15})$$

$$\delta\phi = \sqrt{2}(\delta\theta)_A \psi^A \quad (\text{C16})$$

$$\begin{aligned} \delta\psi^A = & \frac{-1}{2} \gamma_{d=4}^{ij} F_{ij} \epsilon^{AB} (\delta\theta)_B - \sqrt{2} i \sigma^i \overline{(\delta\theta)}^A (D_i \phi) + i [\phi, \phi^\dagger] \epsilon^{AB} (\delta\theta)_B \\ & - i \epsilon^{AB} \left(\begin{array}{cc} D & -\sqrt{2} i F^* \\ \sqrt{2} i F & -D \end{array} \right)_B^C (\delta\theta)_C \\ & + i \epsilon^{AB} \left(\begin{array}{cc} \frac{\eta'' + \eta + \eta'}{2} & i\eta''_+ \\ -i\eta''_- & -\frac{\eta'' + \bar{\eta} - \bar{\eta}'}{2} \end{array} \right)_B^C (\delta\theta)_C, \end{aligned} \quad (\text{C17})$$

where A, B, C denote indices of the $\text{SU}(2)_R$ doublets, $\psi^A = (\chi_4, i\chi_1)$, $(\delta\theta)_A = (i\epsilon_1, -\epsilon_4)$ and $(-\sqrt{2} \text{Im } F, \sqrt{2} \text{Re } F, D)$ the $\text{SU}(2)_R$ triplet auxiliary fields. If $\langle F_{mn} \rangle \neq 0$ for $m, n = 4567$ while other $\langle F_{mn} \rangle$ are zero, then $\eta + \eta' = \eta''$ and $\bar{\eta} - \bar{\eta}' = \eta''$ follow. We can see that the $\eta''(\eta''_{\pm})$, or equivalently the $\langle F_{45} + F_{67} \rangle$ ($\langle F_{56} + F_{47} \pm i(F_{64} + F_{57}) \rangle$), play the role of the Fayet-Iliopoulos D-(F-) term parameters in the $\mathcal{N} = 2$ SUSY gauge theories.

APPENDIX D: UNBROKEN SUSY CONDITION FROM ORIENTIFOLD PLANES

An orientifold is an orbifold where the \mathbf{Z}_2 -flip orbifold group action is accompanied by a flipping between left-mover and right-mover on the string worldsheets. This left-right flipping changes the SUSY generator $\mathcal{Q}_{16+}^{(10)}$ into $\mathcal{Q}_{16+}^{(10)c}$ and vice versa, and this is a crucial difference from the ordinary orbifold. Therefore, the unbroken SUSY condition from the O9-plane is given by

$$\mathcal{Q}^{(10)} = \mathcal{Q}^{(10)c}, \quad (\text{D1})$$

and the condition from the O5-planes by

$$\mathcal{Q}^{(10)} = \exp \left(i\pi (T_{(6)}^{45} - T_{(6)}^{67}) \right) \mathcal{Q}^{(10)c} = -\Gamma_{d=10}^{4567} \mathcal{Q}^{(10)c}. \quad (\text{D2})$$

Note that $\mathcal{Q}^{(10)c}$, not $\mathcal{Q}^{(10)}$, appears in the right-hand sides, contrary to the the case of ordinary orbifolding. Remembering that the SUSY charge satisfies the chirality condition $\mathcal{Q}_{\mathbf{16}_+}^{(10)} = \Gamma_{d=10} \mathcal{Q}_{\mathbf{16}_+}^{(10)}$ and $\mathcal{Q}_{\mathbf{16}_+}^{(10)c} = \Gamma_{d=10} \mathcal{Q}_{\mathbf{16}_+}^{(10)c}$, we can easily see that the above two conditions are equivalent to the unbroken SUSY conditions arising from D9-branes (eq.(45)) and D5-branes (eq.(49)). Therefore, the “ \mathbf{Z}_2 ” orbifoldings associated with the orientifold planes parallel to the D-branes do not cause further breakings of the SUSY.

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TABLES

	$\mathbf{5}^*$	$\mathbf{10}$	$\mathbf{1}$	H	\bar{H}	X_β^α, X_0	Q^k	\bar{Q}_k	Q^6	\bar{Q}_6
\mathbf{Z}_{4R} charge	1	1	1	0	0	2	0	0	2	-2

TABLE I. Charge assignment of the discrete \mathbf{Z}_{4R} symmetry

	$\text{SO}(2)_{45}$	$\text{SO}(2)_{67}$	$\text{SO}(2)_{89}$
$\mathcal{Q}^{(4)1}$	$-1/2$	$-1/2$	$-1/2$
$\chi_1^{\text{U}(5)_{\text{GUT}}}, \chi_1^{\text{U}(3)_{\text{H}}}$	$1/2$	$1/2$	$1/2$
$A_\mu^{\text{U}(5)_{\text{GUT}}}, A_\mu^{\text{U}(3)_{\text{H}}}$	0	0	0
σ, x	1	0	0
σ', x'	0	1	0
σ'', x''	0	0	1
χ_2, \tilde{x}	$1/2$	$-1/2$	$-1/2$
χ_3, \tilde{x}'	$-1/2$	$1/2$	$-1/2$
χ_4, \tilde{x}''	$-1/2$	$-1/2$	$1/2$
q^k, \bar{q}_k	$1/2$	$1/2$	0
$\tilde{q}^k, \tilde{\bar{q}}_k$	0	0	$-1/2$

TABLE II. The field contents arising from open strings on the D3-D7 brane system transform under the space rotational symmetry $\text{SO}(6)_{456789}$. Charges of these fields for the maximal torus of this $\text{SO}(6)_{456789}$ (*i.e.* $\text{SO}(2)_{45} \times \text{SO}(2)_{67} \times \text{SO}(2)_{89}$) are summarized. Charges are determined by observing which open-string modes correspond to those fields. They are combined into chiral multiplets of $\mathcal{N} = 1$ SUSY of $\mathcal{Q}^{(4)1}$ as $\mathcal{W}_\alpha^{\text{U}(5)_{\text{GUT}}} = (\chi_1^{\text{U}(5)_{\text{GUT}}}, F_{ij}^{\text{U}(5)_{\text{GUT}}})$, $\mathcal{W}_\alpha^{\text{U}(3)_{\text{H}}} = (\chi_1^{\text{U}(3)_{\text{H}}}, F_{ij}^{\text{U}(3)_{\text{H}}})$, $\Sigma = (\sigma, \chi_2)$, $X = (x, \tilde{x})$, $\Sigma' = (\sigma', \chi_3)$, $X' = (x', \tilde{x}')$, $\Sigma'' = (\sigma'', \chi_4)$, $X'' = (x'', \tilde{x}'')$, $Q^k = (q^k, \tilde{q}^k)$ and $\bar{Q}_k = (\bar{q}_k, \tilde{\bar{q}}_k)$. Note that the charges of the 2nd-lowest component of all chiral multiplets are $-1/2$ smaller than those of the lowest components. This means that these symmetries can be regarded as R-symmetry.

FIGURES

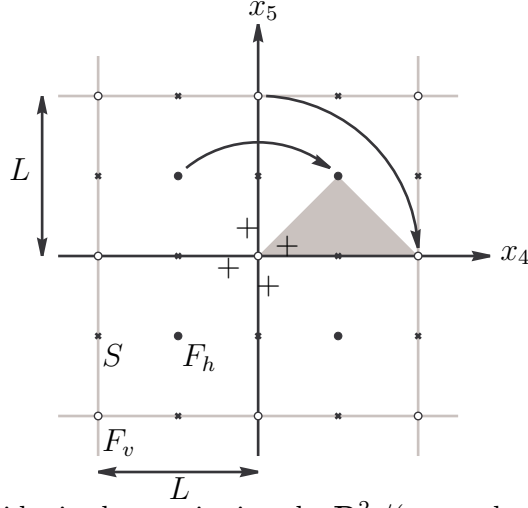


FIG. 1. \mathbf{T}^2 torus we consider in the text is given by $\mathbf{R}^2 / (\text{square lattice})$ shown in this figure. \mathbf{Z}_4 orbifold group action on the x_4 - x_5 plane and the fundamental region of this \mathbf{Z}_4 are also described. There are three distinct singularities among which F_v and F_h are \mathbf{Z}_4 fixed points. Our visible sector 3-brane is located at the \mathbf{Z}_4 fixed point F_v and the hidden sector 3-brane at the other \mathbf{Z}_4 fixed point F_h . S is another singularity in the orbifold which is fixed under $\mathbf{Z}_2 \subset \mathbf{Z}_4$ but not fixed under the whole \mathbf{Z}_4 . '+'s are the location of the hypercolor $\text{U}(3)_\text{H}$ 3-brane and its mirror images under the \mathbf{Z}_4 transformation (their images under $\mathbf{Z} \times \mathbf{Z}$ lattice translation are omitted).